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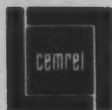
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ABSTRACT

This guide represents the final experimental version of an extended pilot project which was conducted in the United States between 1973 and 1976. The manner of presentation and pedagogical ideas and tools are based on the works of George and Frederique Papy. They are recognized as having introduced colored arrow drawings ("papygrams") and models of our numeration system (the Papy "minicomputer") into the teaching of mathematics at the elementary and secondary level in Belgium. This program follows the "spiral approach." The series emphasizes that a topic may provide children with intuitive leaps which might help them acquire successive pieces of information. Suggestions are presented for obtaining and preparing materials for lessons. Some are available from CSMP. The seven books of the Teacher's Guide for this level are bound into three volumes. This volume contains the General Introduction and the six workbooks. (Author/SA)

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Comprehensive School Mathematics Program
CEMREL, Inc.

CSMP Mathematics for the Intermediate Grades Part I

Experimental Version

4-20601

Teacher's Guide

General Introduction Workbooks

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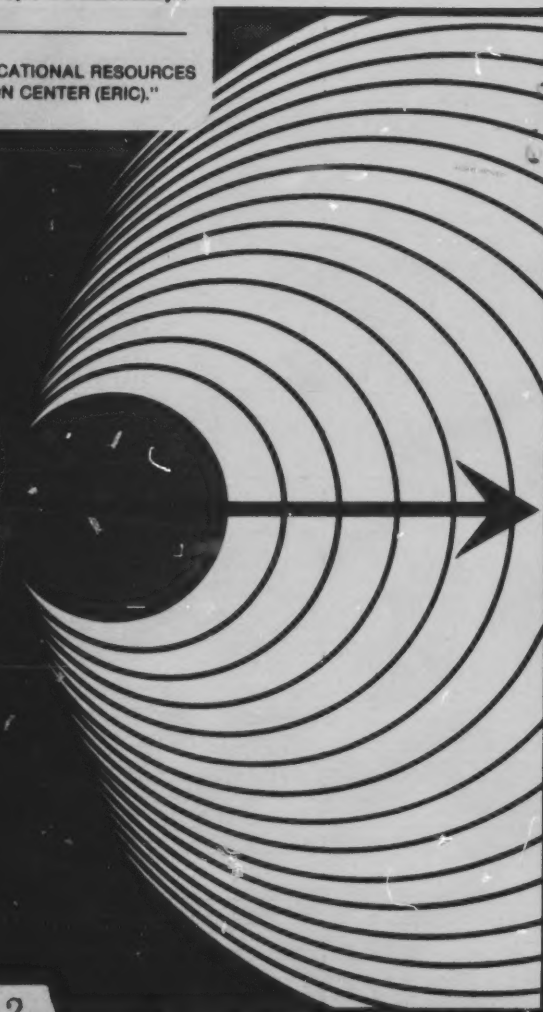
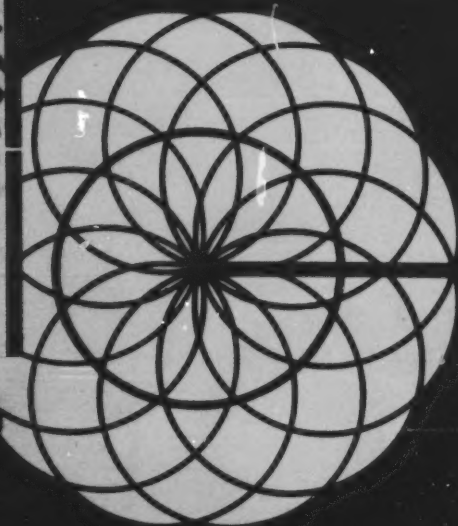
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Comprehensive School Mathematics Program

**Mathematics
for the
Intermediate Grades
Part I**

CEMREL, Inc.

PREFACE

CSMP is firmly committed to the proposition that the study of mathematics should be an interesting, challenging, intellectual and aesthetic experience for all children. It expects the teachers who will teach its curriculum to be similarly committed to that proposition. The CSMP curriculum expects to truly engage children in the study of mathematics. It expects to call upon their curiosity, their intuition, their powers of reasoning, their imagination, their perseverance, and their organizational skills. We believe that the study of mathematics not only calls upon these faculties of children, but will help to develop them. But the curriculum cannot do the job by itself — there must be teachers who share the basic intellectual values built into the curriculum.

There is an almost infinite variety of uses to which mathematics can be, and has been put. Certainly, a reasonable mastery of arithmetic skills is part of the mathematical equipment of a competent, modern adult. Our job as educators is to provide children with a set of mathematical experiences far broader than the acquisition of simple arithmetic skills. It is just as important to expose children to wide variety of ways in which mathematics can help them to formulate and think about problems. We believe further, that the arithmetic skills themselves can be much more successfully taught in the context of a rich and challenging mathematical environment. It is especially important to expose children to a wide variety of mathematical activities in view of the fact that no one can possibly know what career and lifestyle an elementary school child will ultimately want to choose.

It is CSMP's conviction that mathematics is a unified whole and should be learned as such. The organizational basis of the CSMP curriculum is the spiral approach. Thus, the program is sequenced so that students come into contact with each area of content continuously throughout the program. This approach consciously precludes atomizing the content and mastering each bit before continuing to the next. Rather, students encounter repeated exposures to the ideas, building interlocking experiences of ever increasing sophistication.

CSMP believes that no single method of classroom management suffices to meet the needs of every student. Hence, the program is constructed to allow numerous opportunities for whole class participation, small group participation and independent individual study.

The CSMP materials are constructed to maximize teacher-student and student-student interaction. Children begin immediately to investigate mathematical concepts in concrete situations appealing and meaningful to them at their level of experience.

Individualization at this level is achieved through graded series of workbooks integrated into the basic sequence of lessons. Due to the spiral nature of the curriculum each student can engage in a workbook series for a period of from two to four weeks without losing touch with the progress of the class and, at the same time, working his or her way up through the series to the limits of his or her capacity.

Unlike most modern programs, the content is not presented as an artificial structure external to the experience of children and into which they are thrust but rather as an extension of experiences children have encountered in their development, both at the real life and fantasy levels. Children are led through sequences of problem solving experiences presented in game-like as well as story settings. Nonverbal languages give children immediate access to the mathematical ideas and methods necessary not only for solving these problems but also for continually expanding the students' understanding of the mathematical concepts themselves. These languages include:

1. The language of strings. This language mimics the basic method of organizing and collecting data related to problems at hand. That is, it deals with the fundamentally useful and important mathematical notion of sets.
2. The language of arrows. This graphical language models the fundamental process involved in comparing and analyzing sets and operations on them. That is, it deals with the useful and important

mathematical notions of relations and functions including, but not restricted to, permutations of sets, ordering relations and the numerical processes of adding to, subtracting from, multiplying by and sharing equally among.

3. The language of the Papy Minicomputer. The Papy Minicomputer, an extremely effective abacus invented by Georges Papy, models the positional structure of our system of numeration and lends itself to very sophisticated algorithmic processes, including the standard algorithms for the basic numerical operations.

Through these languages the children are able to apply the basic mathematical ideas effectively and creatively to explore the content directly without wading through the cumbersome, sterile terminology and notation which, in most other programs, pervade the early stages of the curriculum and stand as barriers between the child and the content.

The content is learned in an atmosphere of constant connection with applications, from simple story situations in the early grades to challenging applications to nontrivial simulations of real world problems in the upper grades. The emphasis is at all times on a two level approach to learning: understanding the content and its applications, but equally important, developing the techniques and processes of learning the content. It is the latter form of knowledge which gives the power to apply the former.

The Preface above also serves as the Preamble to the Memorandum of Agreement between CEMREL, Inc., and each school district involved in the extended pilot test stage of the CSMP elementary school curriculum.

NOTE ON AUTHORSHIP

"Teacher's Guide to CSMP Mathematics for the Intermediate Grades, Part I" is the product of the concerted efforts of a number of people over several years' time. This edition was produced during the 1975-76 academic year by the following members of the CSMP staff and includes substantial contributions from the staff associates listed below:

Writers:	Burt Kaufman Edward Martin Frederique Papy
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The materials in this guide were developed and tried out with two fourth grade classes at the Jackson Park Elementary School, University City, Missouri. The students, representing the full range of abilities, had been

introduced to the CSMP curriculum in a one semester program using "CSMP Mathematics for the Intermediate Grades, Entry Module I." The entry program prepared these students to begin this first part of the CSMP fourth grade curriculum for veteran students during the second semester of the year.

We are extremely grateful for the daily cooperation and high level of constructive criticism received from Mrs. Linda Levinson and Mrs. Juanester Russell, the regular teachers of the two classes used for developmental purposes. Our greatest motivation for this work was the enthusiasm and steady progress of the students themselves.

General Introduction

Comprehensive School Mathematics Program

**Mathematics
for the
Intermediate Grades
Part I**

General Introduction

CEMREL, Inc.

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1. The CSMP Viewpoint

The CSMP curriculum is based on the belief that mathematics belongs to that part of the school curriculum which deals with ideas rather than that part which stresses techniques. This is not to say that there are no techniques involved in the learning of mathematics. There are, just as there are in the learning of English. But these techniques, just like spelling and grammar in the learning of English, are not ends in themselves; they are important only in so far as they help children to understand and use some area of mathematics.

What are the objects of mathematical study? For us, they include numbers of various kinds, operations on and relationships between those numbers; geometrical figures and their properties; relations and functions (e.g., the $2x$ function, the "I am less than you" relation, etc.) and operations on them.

Among all objects of mathematical study, from the simplest to the most complex, there is one common property: if we separate these objects of study from the real life situations that they model, they are all abstract; that is, they are things of the mind. For this reason, mathematics is, at heart, an intellectual activity. For example, the number 2 represents the phenomenon of twoness, a quality which is present in an endless variety of contexts in life. We can picture 2 by two dots, or by a stick two centimeters long, and these representations are certainly useful in suggesting to our minds what may or may not be true about the number 2. But ultimately the number 2 is accessible to us only through our minds. In the same sense, a geometrical line is neither a pencil line nor a stretched string nor a ray of light, but is an abstraction which may be pictured in many ways.

This is surely an adult point of view. But it is one that should be understood and taken into consideration by those who develop mathematical materials for children and those who teach the materials to children.

Since mathematical objects are ideas, one of the important ways we explore the properties of and relationships among such objects is to think about them. It is for this reason that the growth of the ability to reason plays such an important role in mathematics. We strongly argue, however, that in mathematics the development of the art of reasoning goes hand in hand with the growth of imagination, ingenuity, and intuition. The really important thing at the elementary level is that mathematical objects, their properties and their interrelationships be thought about and discussed in various suggestive and interesting situations.

Seen in this perspective, elementary arithmetic then takes the form of "adventures in the world of numbers." Individual numbers such as 0, 1, 2, 100, $\frac{1}{5}$, -7, $-\frac{4}{9}$, 3.72, -1.006 and so on can assume definite personalities of their own in this world. The teaching of arithmetic shifts its emphasis from an obstacle race for mastery of the standard algorithms to the stimulation of exploring the world of numbers. Skills such as balancing a check-book have their place in real life but are of absolutely no interest to a fourth-grader. It is unrealistic to view such activities as goals of an elementary school mathematics education. Far more genuinely rewarding are the stretching of the powers of imagination and the challenging of the mind. Facility then grows naturally, along with knowledge and understanding. Necessary skills become by-products of such an educational experience.

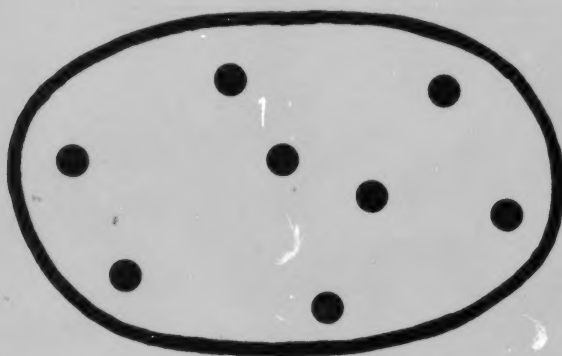
Such a curriculum requires a special kind of pedagogy. The pedagogy employed should include teaching strategies which encourage creativity and allow freedom to explore but still foster sound intellectual habits. Attitudes towards pupil achievement that allow for intellectual and emotional growth and respect the individuality of each student are essential to the successful attainment of the goals of this type of curriculum.

The pedagogy which CSMP has developed in this spirit can be described as a "pedagogy of situations."^{*} The introductions to the various strands of

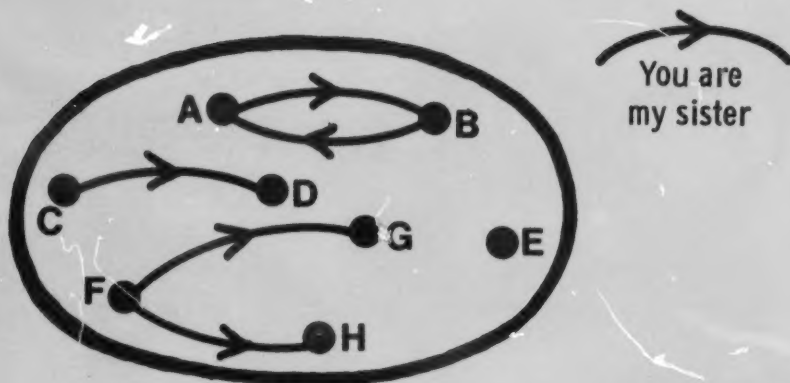
^{*} This phrase is due to the mathematician and psychologist, Caleb Gattegno.

the CSMP Intermediate Grades Program and those to the chapters within them contain many examples which illustrate aspects of this pedagogy. All these examples are taken from the lessons themselves, and a prime source of "pedagogical situations" is the lessons of the program. At the risk of oversimplification, a pedagogy of situations may be described as one which is based on the belief that learning occurs in reaction to the experience of confronting a situation (real, simulated, or imagined) that is rich in consequences, worthy of confrontation, and has genuine intellectual content. For example, we can tell a story about some children in a playground.

As we tell the story, we draw this diagram:



Each child in the playground is represented by exactly one dot, and each dot represents exactly one child. (There is no need to draw pictures of the children; indeed, such pictures would not serve our purposes at all. Even young children are quite able and willing to accept a dot as a symbolic representation for a child. On other days and in different situations they will accept the dot as representing a duck in a pond, or a marble in Johnny's collection, or a number, or whatever else you or they decide.) The children in the playground are playing a game: each child points to his or her sister(s) and says, "You are my sister." As we continue the story we draw arrows as indicated. (We label each dot here with a letter for easy reference as we discuss the situation.) So, for example, C is pointing to D and saying, "You are my sister."



What kinds of comments will young children make when involved in this situation? Here are a few typical responses:

1. There are eight children in the playground.
2. D is C's sister.
3. D is a girl. (Why? Because D is someone's sister.)
4. E has no sisters — at least no sister in the playground.
5. A and B are sisters and therefore both are girls.
6. There are two arrows missing. (Which? G and H are sisters of F and so they should be pointing to each other.)

These two arrows are added to the diagram and the children are now told that all the children that can say "You are my sister" have done so. That is, all the red arrows that belong have been drawn.

7. At this point some child in every group will almost always suggest "C is a boy!" If we ask "How do you know that?", we get an answer that can

be summarized in adult language as: Suppose C were a girl. Then C would be D's sister. But there is no arrow from D to C. So the diagram tells us that C is not D's sister, and hence C is not a girl. So C is a boy.

8. There are children from four different families on the playground.

9. C is D's brother.

The students are now asked to draw arrows for "You are my brother" in a different color. They draw "You are my brother" arrows from D to C, G to F, and H to F.

In this example, the children react to a situation that intrigues them. They grapple with problems of symbolization and the relation of a symbol to the thing it represents. Arguments are given that are close to those which are the stuff of mathematics. Although adults may protest that it is not a "real" situation, it is real to children, who can often involve themselves in and feel at home with "pretend" situations more readily than in the case of many "real" situations. On the basis of several years' experience in a wide variety of classes we can say with confidence that such situations

- involve children at a personal level (they are interested in the situation presented, and sometimes want to dramatize it or devise similar or related situations)
- present an intellectual challenge that is accessible to a broad range of abilities (even the slowest students understand the challenge and can learn something from their experience in confronting it; some may even be able to realize the full learning potential of that challenge)
- provide experiences that have mathematical content. In the example of the "brother-sister situation" above there are two important mathematical activities at work. One is the classification of objects according to certain attributes (brother-sister, boy-girl, same family). More importantly, the experience concerns, at a very

elementary level but in a natural setting, the process of deducing certain conclusions from given data and at the same time realizing the limits of deduction in the situation at hand.

- suggest opportunities for creativity (some children are motivated by such situations to generalize from their experiences or to make conjectures to be tested in similar situations).

The CSMP curriculum fulfills what we consider the fundamental role of mathematics in the elementary school by presenting a large number of varied yet interrelated situations that provide the experiences out of which mathematics grows. Two questions immediately arise. How can the children learn necessary "number facts" via a pedagogy of situations? And how can we "grade the achievement" of a child as he or she reacts to a situation?

Believe it or not, the "number facts" are among the things which children learn almost inadvertently under a pedagogy of situations. Numbers take an almost living form and have personalities of their own. Their interrelations are the subject of many stories in which numbers talk to each other and react to each other. In confronting some of these situations it is necessary to make various calculations, and calculation which serves some higher, more motivating purpose presents nowhere near as many difficulties as does calculation for its own sake. There are further remarks concerning calculation scattered through the later part of this introduction, and also in the introductions that appear in the World of Numbers strand.

As for "grading of achievement," CSMP takes the position that learning often stops when the purpose of a lesson or sequence of lessons is the mere mastery of skills. Every child should learn something from each situation; otherwise it is not a fruitful situation and should be revised or eliminated. But to measure what or how much has been learned in any one situation is probably not possible. Learning is not a linear process. It occurs in quantum jumps that take place at times when you least expect them. It is possible to monitor the progress of children, and you all have your own ways of doing this. But who is to say at what precise time everyone in a class is supposed to have

gained a particular skill or concept from his or her experiences? We comment further on this matter in the next section of this General Introduction.

A more substantive question is: How can we present situations rich in mathematical content before children have the verbal equipment and the computational skills to deal with them? It is presupposed by many people that children must be able to read and to calculate before they can tackle any mathematical situation other than trivial ones. Yet if one analyzes the types of thinking and the few basic concepts underlying elementary school mathematics, one finds that there is nothing esoteric or even difficult about these thinking modes or these concepts. What has been difficult for school children in the past is the language in which these modes of thinking and these concepts are traditionally couched. For some reason, in our natural languages these modes of thinking are difficult to communicate precisely. That is why mathematicians devise languages of their own, languages that minimize the ambiguities of natural languages. Part of the problem in the past has been that elementary school children have found it difficult to deal with such tricky uses of language at a time when they do not yet have a firm command of their natural language.

On the other hand, since the underlying ideas and modes of thinking are inherently simple, there is no reason why very young children should not have the pleasure of such thinking and of exploring mathematical ideas. All that is needed are languages with which to express such ideas and to communicate the thinking that is generated by the ideas. These languages must be very simple, essentially nonverbal, and precise.

CSMP uses three such languages that fit the criteria and that have been highly successful in allowing young people to do the kind of thinking that we described above. We call them the language of strings, the language of arrows, and the language of the Papy Minicomputer.

The first two, as their names imply, are picture languages: brightly colored strings and dots enable precise descriptions of and the accurate recording of

thinking about classifications of objects; colored arrows between pairs of dots stimulate thinking about relations between objects. For detailed descriptions and discussions of these two languages you are referred to the introduction to the Languages of Strings and Arrows strand.

The third language is based on a simple abacus called the "Papy Minicomputer". Using this language, one may represent all decimal numbers (positive or negative) in such a way as to encourage creative thinking about the nature and properties of those numbers. With the support provided by the Minicomputer, the children, even at an early age, have the opportunity to develop excellent mental arithmetic skills and to gain sophisticated insights into the world of numbers. The ability to perform mechanical computation is one of the incidental by-products of such an approach to work with numbers. Many more details on this subject are provided in the introductions that appear throughout the "M" part of the World of Numbers strand.

2. Program Description

a) Description of Materials

The CSMP Intermediate Grades curriculum is designed as a program for students in grades four, five and six. There are six parts to the curriculum, each corresponding to one semester's work. This is Part I of the six. It follows Part II of the CSMP Upper Primary Grades Program.

In addition there are three Entry Modules, one for each grade level. As their names imply, each Entry Module is designed to facilitate transition into the CSMP Elementary School curriculum.

The materials of Part I of the CSMP Intermediate Grades Program require some explanation. This collection of seven books which are directed specifically to you, the teacher, is known as the "Teacher's Guide to CSMP Mathematics for the Intermediate Grades, Part I". (By way of abbreviation, in what follows we shall refer to this collection simply as the "Teacher's Guide".) With the exception of this book, "General Introduction", each

book of the Teacher's Guide addresses itself to a single area of the curriculum, and each of these areas is known as a "strand". The four content strands are as follows:

- The World of Numbers. This strand falls naturally into two parts: one part containing those lessons which according to our suggested schedule should be taught on Mondays (this part is abbreviated "M"); and the other part containing lessons to be taught on Fridays (this part is abbreviated "F"). All worksheets that accompany lessons in this strand (whether from the "M" part or from the "F" part) are identified by the abbreviation "N".
- The Languages of Strings and Arrows. The abbreviation for this strand is "L".
- Geometry and Measurement. The abbreviation for this strand is "G".
- Probability and Statistics. The abbreviation for this strand is "P".

The seven books comprising the "Teacher's Guide" are bound into three volumes, with a separation page inserted between the various books of a given volume. This volume contains both this General Introduction as well as the Workbook strand (which is abbreviated "W"). One of the other two volumes contains the World of Numbers strand (the "M" part and "F" part are each considered as a separate book) and the other volume contains the Languages of Strings and Arrows strand, the Geometry and Measurement strand and the Probability and Statistics strand. Each of these seven books contains its own table of contents and is paginated individually.

The arrangement of material within each content strand is as follows:

- A table of contents. This enables you to locate whatever you need in that particular strand.
- A suggested schedule for the teaching of the lessons in that strand. This provides you with one among the many possible ways of organizing the material in the strand. It also includes for each suggested lesson a list of the materials you will need when you teach

that lesson. See Section 3 of this General Introduction for a discussion of the way in which you should view this suggested schedule.

- A suggested chronological order of the lessons in that strand. This provides any easy, quick reference uncluttered by extraneous information. It shows you in which order the lessons described in the strand will be taught if you follow the suggested schedule.
- An introduction to the strand. This provides you with information explaining why certain material finds its place in that strand, what are the overall goals of study in that strand, and how this material fits in with that which is described in other strands.
- Suggestions for the planning and conducting of lessons in that strand. These include lesson descriptions, suggestions on the use of worksheets and manipulative materials, general remarks relating to certain sequences of lessons, etc.

The suggested lessons are grouped in chapters, all the lessons described in the same chapter having some particular feature(s) in common. Because of this commonality of purpose, it is possible to precede each chapter by an introduction which indicates where the lessons in that chapter are going and on occasion gives a few more details concerning the mathematical underpinnings of the lessons described in the chapter than are contained in the lesson descriptions themselves.

Many of the suggested lessons require student worksheets for individual work. Classroom sets of these worksheets are printed and packaged separately from this guide. However, an answer key for each of the worksheets is included in the guide. Worksheets are numbered consecutively within each strand and their answer keys can be found at the end of the lessons for which the worksheets are needed. For example, an answer key for Worksheet P03 is at the end of Section 1.3 of strand P since it is suggested that this worksheet be used with the lesson described in that section.

In order to accommodate the various ability levels in a given class, the worksheets generally are written at three levels of difficulty denoted by stars. "*" represents an easy worksheet, "***" represents an average level worksheet, and "****" represents a difficult level worksheet. All students should begin with the "*" level worksheets for a given lesson or sequence of lessons and proceed as far as possible through the other two levels. In classes containing the usual range of abilities, most of the students should succeed with the one-star worksheets; the two-star worksheets should be appropriate for about two-thirds of the class; and approximately one-third of the students will find the three-star worksheets challenging and enjoyable to do. No one should ever be required to attempt all the worksheets provided for a given lesson. Each child should progress as far as his or her abilities allow.

In strand P there are a few pages included with the worksheets that do not really belong in this category. These are printed materials needed for various lessons in that strand and which, due to practical considerations, were printed and packaged with the worksheets. These particular pages are numbered in the worksheet sequence but do not contain any stars to designate difficulty levels.

There are various other items the use of which is (or could be) woven into the program. Among them are the following: various manipulative materials such as translators, spinners, compasses, and so on, which are described in more detail in the lessons in which they are used; several workbooks, storybooks and story-workbooks, which are described in more detail in Sections 2 (b) and (e) of this General Introduction, and in the introduction to the Workbook strand; a book entitled "Mathematical Play Therapy", which includes a collection of suggested small group activities that could be instrumental in stimulating an open and positive attitude to mathematics on the part of those of your students who find the subject difficult. See the introduction to "Mathematical Play Therapy" for suggestions as to its use in the program.

b) Role and Use of Workbooks

It is suggested that one day each week be set aside for students to do individual work in designated workbooks. The workbooks are designed to review ideas encountered in the content strands and to apply these ideas to new situations posed as stories. Many of the workbooks are written at more than one level of difficulty in order to accommodate differences in pupils' abilities and interests.

More detailed descriptions of the workbooks, their purposes, and answer keys are given in the Workbook strand. There you will also find suggestions for keeping records of pupil progress.

c) The Spiral Development

Since the learning process occurs through many interrelated experiences, we have developed this curriculum so that no single lesson is an end in itself. It is neither intended nor expected that each child will meet the full challenge of a particular situation in any given lesson. Furthermore, we have found that it is most effective to vary the situations and strands from day to day rather than to continue with one type of situation or piece of content until so-called mastery has occurred. Thus, we suggest that the lessons be scheduled so that a different topic is studied each day of the week. In a typical week, for example, your class might work with numerical games one day, probability the next day, workbooks the third day, geometry the fourth day, and numerical relations the fifth day.

This spiral development, in which a child experiences each of several ideas a little at a time and then proceeds through increasing levels of sophistication as the situations become more challenging, reflects the fact that learning is not a linear process. Not only do different children learn at different times and different rates, but even on an individual level learning can occur in quantum leaps that take place at times when you least expect them. The spiral development gives each child a new chance to catch on to an idea at each turn of the spiral.

Of course, acceptance of the CSMP pedagogy requires a certain act of faith on your part. If you have been accustomed to staying with a given topic until everybody (or nearly everybody) in the class knows it "cold," then you may be initially very reluctant to leave a lesson at the end of the time suggested, believing that several children "haven't got it yet." By keeping the spiral idea in mind, you may become more comfortable with these time schedules. Although it may appear to you that no learning is taking place in a sequence of lessons, you may be surprised that suddenly even your slowest students are understanding the ideas when they encounter them later in the spiral. Whereas with a mastery approach, presented in a purely individualized mode, slow students often spend most of the school year trying to "master" low level skills (with generally poor results), the spiral approach gives these students the opportunity to benefit from the richness of the situations presented throughout the curriculum. In this manner children are allowed to become involved in interesting and enjoyable situations which are modeled by important mathematical notions. They are not deprived of a variety of experiences just because they haven't mastered every bit and piece along the way. Therefore, in the long run, their interest in, understanding of, and even facility with mathematics has a much better chance to develop.

d) Evaluation

On the basis of the preceding discussion, it should come as no surprise, as stated earlier that CSMP takes the position that learning often stops when the purpose of a lesson or sequence of lessons is the mere mastery of skills. Furthermore, we do not believe it is desirable or reasonable to attempt to assess student mastery of skills from day to day or even from week to week. As we said before, learning has a habit of occurring when you least expect it, so how does one determine when everyone in a class should have mastered a specific skill or attained understanding of a particular concept as a result of his or her experiences? We know of no method for measuring what or how much has been learned from any one situation except in the most trivial cases. On the other hand, teachers, parents, and even the children

themselves can benefit from some continuous monitoring process that records involvement in activities and assesses progress through the program. Indeed, it is possible to monitor your students' progress without contradicting the statements made above; all of you have your own ways of doing this. Such a process is suggested in the introduction to the Workbook strand. Of course, you should also continue to use any monitoring devices with which you feel comfortable.

As far as traditional computational skills are concerned, the standardized test data collected during the extended pilot test stage of the development of this curriculum indicate that CSMP classes generally do better than average on the items which test "basic" skills.

About 75% of the lessons for the CSMP Intermediate Grades Program involve numerical concepts. The situations do not involve just the whole numbers (as is generally the case at this level), but introduce the students to many other citizens of the world of numbers at as early a stage as possible. You should not infer from this that your students are expected to have mastered the arithmetic of the whole numbers, the integers, the decimal numbers, and the rational numbers before they are ready for Part II of the CSMP Intermediate Grades Program. On the contrary, it means that the students will become more and more friendly and at ease with the various types of numbers and will develop their own methods for calculating with them long before the standard mechanical algorithms are introduced.

It is essential that the creative freedom of the student not be impaired by a premature emphasis on the standard algorithms for adding, subtracting, multiplying and dividing. We believe that many of the present computational difficulties experienced by elementary and junior high school students were caused by their spending an inordinate amount of time trying to master the algorithms too early in their mathematical education. This shuts the students off from other experiences necessary for their mathematical growth. Students need plenty of time to explore the world of numbers. Mastering an algorithm should be the last step of this exploration, a sort of natural synthesis of previous experience. Furthermore, as electronic hand-calculators

become available to nearly everyone, algorithmic mastery as a prerequisite to other numerical experiences should become less important.

We therefore strongly urge you to follow the suggested schedule of lessons, teaching the algorithms as they come, but not giving them more emphasis than they deserve. Skill in mental arithmetic and estimation, however, is extremely important in our opinion and should be promoted at every opportunity.

e) Role and Use of Storybooks and Story-Workbooks

Among the instructional materials for the CSMP Elementary School curriculum is a collection of mathematical storybooks, *STORIES BY FREDERIQUE*. This collection consists of 24 stories for young people, each of which is presented in the format of a newspaper for children. Nearly every story is printed in color on tabloid size ($17\frac{1}{2} \times 11\frac{1}{2}$)^{*} newsprint and is intended to appeal to youngsters in the 5-14 age range. The stories are completely independent of each other and need no external explanation. They may be used for individual or collective reading, at home or at school.

Each of the stories is portrayed by means of the mathematical languages of arrows, dots and strings which reinforce the ideas of the story. Three of the stories (*TWO BY TWO*, *THE WEIRD STORY OF 24*, and *SUMMER SCHOOL IN THE OLD DAYS*) also use the language of the Minicomputer. The amazing versatility of these nonverbal languages is exploited to the full.

The young readers are personally involved in each story. They have to answer questions and use their imaginations to invent additional adventures for the characters in the story. They learn that there is room in the make-believe world of mathematics for exercising the aesthetic sense and for expressing emotions.

* Six of the stories are printed on $8\frac{1}{2} \times 11$ " newsprint rather than the tabloid size.

Twelve of the stories are intended for very young children (5 to 8 years of age). Here the focus is on the anecdotal aspect of the story. One of the goals of these twelve books is to make the children familiar with nonverbal mathematical languages and to leave them with the impression that these languages are as natural and useful as the usual language.

Nine of the remaining stories concern numbers and their relationship with the world. These particular stories are about a boy and some of his friends who just happen to be numbers. The personification of numbers introduces the "games" of mathematics into one of the most important domains of childhood — the world of fantasy. Free from the usual constraints so often placed upon them, the numbers invent marvelous games and dance to the tunes of vividly colored arrows. These games are never frivolous. There are rules to be followed and as a result, orderly patterns emerge as the numbers play or dance.

Of the remaining three books, two are detective stories, introducing some interesting mathematical problems, and one emphasizes the various and very different aspects of a simple straight line.

The titles of the books in the collection and the intended age range for each story are as follows:

Ages 5 to 8

THE PLAYFUL NUMBERS

81 ROSES

I AM A VERY HAPPY BOY

ONE OUT OF SEVEN

THE HAPPY PUPPET

TWO BY TWO

THE LITTLE DREAMER

THE LITTLE DONKEY

THE MAGIC BOX

THE BABY IS BORN

THE WEIRD STORY OF 24

THE OLD SHOEMAKER

Ages 8 to 12

WHERE'S MY NOSE?

SINGING FRIENDS

DANCING FRIENDS

THE LIVING LINES

I AM NOT MY NAME

NABU WINS AN AWARD

THE SQUARE TRAP

SUMMER SCHOOL IN THE OLD DAYS

Ages 10 to 14

THE HIDDEN TREASURE

A VERY STRANGE NEIGHBORHOOD

ELECTION IN THE NUMBER WORLD

A VALENTINE MYSTERY

In Part I of the Intermediate Grades Program, there are three lessons explicitly involving one of these books: THE SQUARE TRAP. In addition two copies each of the following storybooks are provided: I AM NOT MY NAME, THE HAPPY PUPPET, THE MAGIC BOX, THE LIVING LINES, THE LITTLE DONKEY, WHERE'S MY NOSE?, DANCING FRIENDS, SINGING FRIENDS, and TWO BY TWO. They should be put in the class library for students to read whenever they like. These are books that were in the third grade program and that students may enjoy rereading from time to time. If Part I is being used in the second semester, following Entry Module I, the last five books are not included with the Part I materials since they were provided with the entry module.

Besides the collection, STORIES BY FREDERIQUE, another collection, THE CSMP LIBRARY, has been developed as part of the instructional materials for the CSMP Elementary School Program. This is a series of story-workbooks which combines the spirit of the storybooks and the workbooks into a single integrated experience for the children, engaging them in reading about and doing mathematics in stimulating, challenging story situations. Four of these, A BOOK ABOUT ME, HALLOWEEN PUZZLES, SUMMER CAMP and NOT TOO CLOSE are incorporated into the Workbook strand. One, WHO ARE WE? is utilized in the World of Numbers strand.

f) Role and Use of Hand-Calculators

The electronic hand-calculator, if used creatively, can have a remarkably beneficial effect on the students' interest in and understanding of arithmetic, and its use can be a contributing factor in the improvement of their capabilities in that area. The reasons for this are many and varied, but among them are certainly the following:

- By removing the burden of mechanical computation from the children's shoulders, the calculator frees them to concentrate on the thinking that is a necessary part of the solution to any meaningful problem.
- The instant feedback provided by the calculator enables the children to indulge in making and testing conjectures, an area into which they would not otherwise venture because the quantity of computation involved would be too forbidding.

This pedagogical tool is particularly useful in the introduction of new numerical notions such as negative integers, decimal numbers, multiples of a given whole number and so on, because by means of suitably chosen, rich situations, these concepts can be made to arise in a natural way.

Two rechargeable, battery powered hand-calculators are included with the materials for Part I. One recharger unit is also included and each hand-calculator should be recharged every other day. The hand-calculators also work on alternating current.

While the two included hand-calculators are adequate for the lessons in Part I, the ideal situation would be for each student to have one. Encourage the students to bring calculators from home. The hand-calculators that

* The Part I material requires two hand-calculators in each class. If you already have calculators, classroom kits may be ordered without them. Be sure to check your calculators for algebraic logic and constant mode.

come with the materials have algebraic logic and function keys which are permanently in a constant mode. You should check to be sure that any calculators brought in by students have these characteristics as follows:

- Algebraic logic. This means that the calculator responds to instructions given in the order in which they are usually written. To check that this is the case, press $\boxed{2} \dots \boxed{+} \dots \boxed{3} \dots \boxed{=}$. If you have "5" on the display, then all is well.
- Constant mode. Some calculators have a switch so that the calculator can be set either in a constant mode or in a chain mode. If that is not the case with your calculator, then check to see whether or not both $\boxed{+}$ and $\boxed{-}$ operate in a constant mode, as follows. Press $\boxed{0} \dots \boxed{+} \dots \boxed{2} \dots \boxed{=} \dots \boxed{=} \dots \boxed{=}$... and so on. If the number on the display is increasing by two at each depression of $\boxed{=}$, then $\boxed{+}$ is operating in a constant mode. Check the constant mode operation of $\boxed{-}$ in a similar fashion.

While it would be possible to use calculators without these characteristics in the lessons, the adjustments that would be required in the instructions make the calculators unsuitable.

3. Guidelines for Organizing the Program

The content of the lessons in Part I is designed to afford you the utmost flexibility in teaching the program successfully. It is a truism to say that your students this year are very different, both individually and as a class, from the students you taught last year, and indeed from other students in any other CSMP class. And your own teaching style and interests are probably very different from those of your colleagues. It is difficult for those of us who develop curricula to guess correctly as to which topics will be very easy for your students and which lessons will require more instructional time. You are the only one who will be able to judge that as you begin using the materials. This program is not meant to be a static sequence of exercises to be followed rigidly; the lessons in this guide should be

regarded as suggestions; they simply reflect our own experience teaching the materials. You and your students will make the program dynamic as together you explore the mathematical ideas and follow your own interests and inclinations.

If this is the first time you are teaching a CSMP class and you would like specific suggestions for organizing the lessons, we offer some guidelines and various suggested schedules of lessons. Please remember that our suggested sequence is but one of many that could have been assembled. You may wish to follow it at the beginning of the school year and then deviate from it as the needs of your students may dictate.

The master schedule which is included at the end of this section was developed for a hypothetical fourth grade class of average ability, and it illustrates some important ideas about organizing the program. You will notice that each day of the week is assigned a different strand. This was done to carry out the idea of a spiral curriculum. If you organize your own schedule, it is best to follow a one-day, one-strand pattern rather than to spend two or three full weeks, for example, "covering" one particular topic such as composition of relations or decimal numbers. Interchange the strands and the days of the week as you please. Two days each week should be devoted to the World of Numbers strand (one from the "M" part and one from the "F" part) and one day each week to the Workbook strand. Strands P, G and L should share the other two days.

Within each strand (or part of a strand) we provide a more detailed schedule aimed specifically at that strand (or part of a strand). In the case of Strands P, G, L which share two days of the week, we provide a schedule which shows how they spiral among themselves. But the copy of this schedule which appears in each strand book has the lessons of the other two strands shaded over so that only the lessons of the strand itself stand out. In each schedule, each block represents one full period of approximately 40-45 minutes' duration. It may very well be, however, that you and your class will spend more than one period on some lessons. Exercise

your own professional judgment in deciding when to stop one particular lesson and when to take it up again. Just be sure that you do not insist that every single child understands every single aspect of a lesson before you go on. It is better to stop a lesson at a point when student interest is high and a problem remains unsolved than to teach so methodically that everyone in the class understands everything, but is completely bored.

Inside each block of the detailed strand schedules, the lesson of the day is designated in various ways. First, there is listed a number which indicates its suggested chronological order among the lessons in that strand. (Hence, "5P" indicates that the lesson in question is suggested as the fifth lesson in Strand P.) This identifying number also appears in the upper outside corner of each page in the description of that lesson, thus enabling a lesson to be located simply by flipping through the pages of the strand book. Second, you are told on which page of the strand book the description of the lesson in question begins. Third, as a double-check, you are told the section number and title of the lesson. Next to each block in each schedule there is a list of the materials you and the children will need in order for you to be able to teach the corresponding lesson. In these materials lists we do not include such items as pencils, paper, and colored chalk, which are required in almost every lesson.

From time to time a free day appears in the schedule. These days should give you opportunities to do many kinds of things. You can review lessons, complete lessons not entirely finished, or extend ideas in lessons already completed. The children can continue working on their workbooks, or finish uncompleted worksheets. Children can gain more experience with the manipulative materials in the program, such as the compasses or the spinners. You and the class can read or reread the mathematical storybooks and make up other stories which use the languages in the CSMP program. You can do additional exercises in mental arithmetic. You can teach topics which traditionally have been placed in the mathematics curriculum but which are not included in this program.

Please bear in mind that the model schedule was designed for a hypothetical fourth grade class with the usual range of abilities. In order to be successful with the program, you should adjust the schedule to the ability, maturity, and interests of your class. For example, you may wish to extend a lesson or a workbook series for several additional periods beyond those scheduled.

**SUGGESTED
SCHEDULE
OF
LESSONS**

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART I

WEEK	MONDAYS The World of Numbers	TUESDAYS Lang. of Strings and Arrows Probability and Statistics Geometry and Measurement
1	M1.1 Transforming a Number and Arrow Pictures #1	L1.1 The String Game with A - Blocks
2	M1.2 Transforming a Number and Arrow Pictures #2	L1.2 The String Game with Numbers: Multiples and Order
3	M2.1 Halving and Decimal Numbers	L2.1 Logical Thinking #1
4	M3.1 Negative Integers on the Number Line and on the Minicomputer	L2.2 Logical Thinking #2
5	M1.3 Introduction to the Mini- computer Golf Game	G2.1 Circles #1
6	FREE DAY	G2.2 Circles #2
7	M4.1 Multiplication Prob- lems #1	G2.3 Circles #3
8	M3.2 Minicomputer Golf with Positive and Negative Checkers #1	FREE DAY
9	M2.2 Dollars, Dimes and and Pennies	G1.4 Area Problems

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART I

WEDNESDAYS Workbooks	THURSDAYS Lang. of Strings and Arrows Probability and Statistics Geometry and Measurement	FRIDAYS The World of Numbers
A Book About Me	G1.1 Comparisons of Lengths and Areas	F1.1 Tag the Arrow Pictures
A Book About Me	G1.2 Introduction to Milli- meters	F1.2 Match the Tags #1
A Book About Me	G1.3 Square Regions	F2.1 Composition of Func- tions #1
Collection of Problems 1 Tim and I	FREE DAY	F3.1 Guess My Rule #1
Collection of Problems 1 Tim and I	P1.1 Same or Different #1	F4.1 Ten Times #1
Collection of Problems 1 Tim and I	P1.2 Same or Different #2	F5.1 Find the Secret Number
Halloween Puzzles	P1.3 Same or Different #3	F4.2 Ten Times #2
Halloween Puzzles	P1.4 Same or Different #4	F2.2 Composition of Func- tions #2
Halloween Puzzles	P1.5 Same or Different #5	F6.1 The Relation, "Is A Divisor Of" #1

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART I

WEEK	MONDAYS The World of Numbers	TUESDAYS Lang. of Strings and Arrows Probability and Statistics Geometry and Measurement
10	M5.1 Fractions #1	L3.1 Composition Games #1
11	M4.2 Multiplication Problems #2	L3.2 Composition Games #2
12	M5.2 Fractions #2	L3.3 Composition Games #3
13	M6.1 Division Problems #1	L3.4 Composition Games #4
14	M5.3 Fractions #3	FREE DAY
15	M3.3 Minicomputer Golf with Positive and Negative Checkers #2	FREE DAY
16	M2.3 Assorted Problems	P2.1 "The Square Trap" #1
17	M4.3 Multiplication Problems #3	P2.2 "The Square Trap" #2
18	M6.2 Division Problems #2	P2.3 "The Square Trap" #3

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART I

WEDNESDAYS Workbooks	THURSDAYS Lang. of Strings and Arrows Probability and Statistics Geometry and Measurement	FRIDAYS The World of Numbers
Summer Camp	FREE DAY	F5.2 "Who Are We?"
Summer Camp	L2.3 Logical Thinking #3	F1.3 Match the Tags #2
Collection of Problems 2	L2.4 Logical Thinking #4	FREE DAY
Collection of Problems 2	FREE DAY	F6.2 The Relation, "Is A Divisor Of" #2
Collection of Problems 2	G2.4 Circles #4	F5.3 Nlx; Plf and Paf
Not Too Close	G2.5 The Triangle Inequality	F3.2 Guess My Rule #2
Not Too Close	L1.3 The String Game with Numbers: Multiples, Divisors and Order #1	F1.4 Find the Missing Arrows
Not Too Close	L1.4 The String Game with Numbers: Multiples, Divisors and Order #2	F5.4 Nick and Nack
FREE DAY	L1.5 The String Game with Numbers: Multiples, Divisors and Order #3	F2.3 Composition of Func- tions #3

4. Commonly Asked Questions about the CSMP Program — and Suggested Answers.

As you teach the CSMP curriculum (or even as prepare to teach it) there will surely be some basic questions that come to your mind. They are the kind of questions that not only you as a teacher, but also parents and supervisors, will ask about any curriculum that appears to be different from the usual program.

The following questions and answers are listed as an aid to you in understanding our curriculum and in responding to those who request information about it.

1. Q: For what sort of student is the CSMP elementary curriculum designed?

A: It is designed for all children in the schools. Each learning situation is designed to involve all students at their own levels in such a way that the slowest can succeed in dealing with some aspects of the situation while the fastest find aspects of the situation that challenge them. In fact, it has been found that such a "pedagogy of situations" (see Question 2) is particularly effective with the so-called slow learners, and recent work suggests it might be effective also with educable mentally handicapped children.

2. Q: Briefly, how does CSMP differ from traditional mathematics program?

A: The core content of CSMP is not revolutionary. The program aims to inculcate notions and facts which have always been considered important in mathematics. The major differences are in method. Traditional teaching takes pride in disciplining the student to be submitted to the usual algorithms. With CSMP the student is

involved in real mathematical activity, with understanding at his or her level, so as to master the situation.

For that purpose CSMP uses new teaching aids which are essentially nonverbal languages and which are themselves both pure and applied mathematics. These mathematical objects; graphs, arrows, etc., are used as aids rather than as ends in themselves.

A pedagogy of situations is employed to permit the pupil to participate with creative freedom in the learning process. The student is faced with situations under which, by spontaneous reaction, he or she is led to an important idea in mathematics. Thereby, the child learns to mathematize situations, which is of fundamental importance whenever mathematics is applied.

CSMP also makes use of a spiral approach rather than the mastery approach found in most mathematics curricula. (See Question 13.)

3. Q: Isn't this the "new math"?

A: There is a current popular belief that all school mathematics today is either "new math" or "old math". Neither of these labels is well defined. "New math" usually denotes the programs that heavily emphasize the teaching of sets; introduce technical mathematical language for concepts such as "commutativity", "associativity", "distributivity", and so on; and that teach "concepts" before "skills". The "old math" refers to programs of the "good old days": memorization of the multiplication tables and complete emphasis on paper and pencil drill in calculations.

The CSMP curriculum is definitely not the "new math" nor the "old" by any definition. It uses no technical language, yet achieves the understandings sought by the "new math". It uses no repetitive drills, yet elicits the ability to calculate stressed by the "old".

But these achievements are just by-products of a more pervasive goal: to help children use their curiosity, their intuition, their powers of reasoning to formulate and think about interesting problems and situations.

4. Q: Is the student really studying mathematics in the CSMP curriculum?

A: Mathematics has a subject matter which involves much more than calculation. CSMP believes that the number system is very important and that it is essential to introduce the pupil to the world of numbers. However, the intelligibility of that world is not essentially a matter of computation. The teaching aids of CSMP give concrete support to some important internal relations in the world of numbers as well as some logical reasoning. The deep understanding of mathematics is not always verbal but is very often pictorial and CSMP tries to respect that aspect.

5. Q: Why use the languages of strings, arrows, and the Papy Minicomputer?

A: It is surprising that the language of strings was not used earlier and more extensively in the teaching of mathematics. Mathematicians have used strings for a very long time and they support notions so fundamental and natural that they go back to Aristotle. The scope of the string diagrams is broader than mathematics, the string diagram being a marvelous instrument for logical thinking. String diagrams permit the understanding of a situation at a glance.

Arrow diagrams are probably the most important teaching aid in CSMP. They are used everywhere in our daily lives; signals, advertisements, directions, and so on. Why not use them in teaching mathematics? Arrows are a pictorial way to represent relations. Relations have always been important in mathematics, but in the

past seemed to be something rather abstract, elusive, and mysterious. With arrows they become vivid, concrete, and easy to understand.

The Minicomputer allows very young children to become familiar with numbers and operations on them (especially large numbers and the more difficult calculations) before they are able to work with them on paper. Later the Minicomputer is used as a vehicle for posing situations involving mental arithmetic and for examining the workings of the arithmetic operations. It provides the experience necessary so that rather than memorizing an algorithm and applying it to situations, the situations give rise to the algorithm.

6. Q: If the students use the Minicomputer, will they be able to calculate without it?

A: This question suggests that the only purpose of the Minicomputer in the CSMP curriculum is to perform calculations. To be sure, the Minicomputer does enable very young children to deal with "large" numbers and perform relatively difficult calculations long before they can write the paper and pencil algorithms for the calculations. But very soon these experiences with the Minicomputer suggest how the paper and pencil algorithms come about, and from that moment the Minicomputer is no longer regarded as a calculating tool. Instead, for the remainder of the curriculum, the Minicomputer assumes its primary roles: as a source of very interesting problems and games, as an effective vehicle for stimulating mental arithmetic, as a teaching device for introducing more advanced topics such as the negative numbers and the decimal numbers, and, in general, as an alternate representation of numbers.

7. Q: What is the CSMP position on hand-calculators and what role do they play in the program?

A: Hand-calculators should be a part of any mathematics curriculum. As more and more children have hand-calculators, these tools will become incorporated in the elementary and secondary curricula in a natural way with uses very similar to those of the Minicomputer. In some cases they may be used as a tool for mechanical computation where the difficulty of the calculation might turn an otherwise interesting problem into an exercise in drudgery. In other cases they may be used as the context for posing situations leading to the discovery of important mathematical properties or providing opportunities for interesting exercises in mental arithmetic.

Hand-calculators are of particular benefit to "slow" students. Many slow learners have mechanical difficulties rather than problems in thinking about the situation. The hand-calculator alleviates this problem.

Some lessons in the CSMP curriculum require only the two calculators which have been provided, while others suggest that each student have one. Lessons of this latter sort, however, also include suggestions on how they may be modified since not every student may have a hand-calculator. In either situation, students with access to hand-calculators should always be encouraged to bring them to class.

8. Q: Why is there a lack of drill and little emphasis on "skills"?

A: CSMP holds that training for rote skill should never precede understanding. The avowed purpose of drill in arithmetic is to acquire the "skills", that is, the skills of calculation. Certain numerical skills are essential, but, unfortunately for school

children, the usual drill techniques for acquiring them are dull, stultifying and sometimes even counterproductive. The CSMP curriculum appears to place little emphasis on "skills" because it does not resort to drill. But as a matter of fact, the essential skills of both mental and written calculation are learned almost inadvertently in response to the many situations where the ability to calculate is needed in order to respond to the situation: to win the number game, to identify the secret number of the detective story, or to build a number road. Always, calculation is considered as a tool for doing interesting things, rather than a chore done for its own sake.

9. Q: How do students learn to use mathematics in practical situations?

A: For students of elementary school age, a situation that intrigues them and catches their imagination or their curiosity will certainly motivate them to learn whatever is necessary in order to deal with the situation. But situations that adults consider "practical" very often are outside the realm of interest of school children and do not provide motivation for learning. CSMP has found that students will indeed learn to use mathematics in situations they consider worthy of their attention. As they approach adulthood they will soon consider "practical" situations to be most worthy of their attention, but in the meanwhile, they are more interested in games, fantasy, puzzles, stories, and so on.

10. Q: Why are we teaching probability?

A: Next to arithmetic, probability has the most direct application to life in such areas as economics, meteorology, data processing, insurance, most of the natural sciences, and most of the social sciences. Probability differs from many areas of mathematics in that it is applied directly. One does not need to combine knowledge of probability with any deep theoretical knowledge in order

to apply it to "real world" problems. One is exposed to statistics nearly every day of one's life, and the better developed one's intuitive sense of probabilistic situations is, the less likely one is to be "duped" by statistics.

11. Q: How are telling time, money, and the metric system treated?

A: Telling time is not included in the CSMP program. It is our belief that telling time is not a part of mathematics, although it is frequently included in the mathematics curriculum. We do not feel that we could improve on the methods currently used, and, for this reason, we urge you to teach this topic using those methods and materials that have proved successful for you in the past.

The topic of money is first introduced in the CSMP curriculum in lessons involving combinatorics. It is treated again in the Upper Primary and Intermediate Grades as a support for the teaching of the decimal numbers. Should the students need more exposure to this topic, teachers should feel free to develop supplementary materials in order to provide extra practice.

The metric system is introduced in the second and third grades, and provides all standardized units of measure for the Geometry and Measurement strand. We recommend that you do not ask the students to convert from the English to the metric system or vice versa (inches to centimeters, yards to meters, Celsius to Fahrenheit, grams to ounces, and so on). Since the United States has not yet "gone metric", your students will still need to be familiar with the English system. We would suggest that you use whatever materials and methods you have found successful in the past in teaching this topic, but we repeat our recommendation that you not ask the students to convert from one system to the other.

12. Q: What is the role of storybooks and story-workbooks in the program?

A: The storybooks, "Stories by Frederique" are a collection of 24 stories for young people, each of which is presented in the format of a newspaper for children. They are intended to appeal to children in the 5-14 age bracket. The stories are completely independent of each other and need no external explanation. They may be used for individual or collective reading, at home or at school.

The story-workbooks, as their name implies, are a cross between storybook and workbook. They present more problems than the storybooks for the students to solve, but within a story setting. The story-workbooks comprise the collection called "The CSMP Library".

Each of the stories is portrayed by means of the mathematical languages of arrows, dots, and strings which reinforce the ideas of the stories. The young readers are personally involved in each story. They have to answer questions and use their imaginations to invent additional adventures for the characters in the story. They learn that there is room in the make-believe world of mathematics for exercising the aesthetic sense and for expressing emotions.

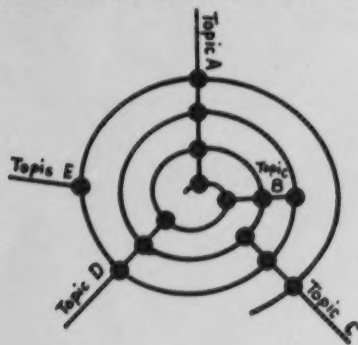
Among the goals of the various storybooks and story-workbooks are the following: to make the children familiar with nonverbal mathematical languages and to leave them with the impression that these languages are as natural and useful as the usual language; to introduce some mathematical games; to present some detective stories which introduce some interesting mathematical problems; to provide reinforcement for all students and especially for the slower learners.

13. Q: What should I do if some of the students don't fully understand a lesson?

A: Much of the following appeared in section 2c but is repeated here for emphasis and your convenience.

Since the learning process occurs through many interrelated experiences, we have developed this curriculum so that no single lesson is an end in itself. It is neither intended nor expected that each child will meet the full challenge of a particular situation in any given lesson. Furthermore, we have found that it is most effective to vary the situations and topics from day to day rather than to continue with one type of situation or piece of content until so-called mastery has occurred. Students at this age do not seem to enjoy staying on the same topic for a very long time, but prefer to be involved in a variety of situations. Thus, the lessons are scheduled so that different topics will be studied during each week and similar situations appear again and again at different times and levels in a spiral development.

If a spiral curriculum seems strange at first, consider the following illustration: Imagine an academic subject that could be divided into five subtopics. Then imagine a child's progress being represented by a spiral, starting at the center and traveling outwards in the direction of increasing degrees of maturity and sophistication. In the diagram, an intersection point between the spiral and a topic line indicates a lesson on that topic:



The child is first introduced to Topic A, then to B, then to D, followed by a slightly more sophisticated lesson A, and so on. Topics C and E are not introduced until a little later in the spiral, while Topic B may be of a more terminal nature and is considered completed after a few lessons.

This spiral development, in which a child experiences each of several ideas a little at a time and then proceeds through increasing levels of sophistication as the situations become more challenging, is wholly consistent with CSMP's pedagogy of situations.

Acceptance of the CSMP pedagogy requires a certain act of faith on your part. If you have been accustomed to staying with a given lesson until everybody (or nearly everybody) in the class knows it "cold," then you may be initially very reluctant to leave a lesson at the end of the time suggested, believing that several children "haven't got it yet." By keeping the spiral idea in mind, you may become more comfortable with these time schedules. Although it may appear to you that no learning is taking place in a sequence of lessons, you may be surprised that suddenly even your slower students are understanding the ideas when they encounter them later in the spiral. Whereas with a mastery approach, presented in a purely individualized mode, slow students often linger most of the school year trying to "master" low level skills (with generally poor results), the spiral approach gives these students the opportunity to benefit from the richness of the situations presented throughout the curriculum. In this manner children are allowed to become involved in interesting and enjoyable situations which are modeled by important mathematical notions. They are not deprived of a variety of experiences just because they haven't mastered every bit and piece along the way.

14. Q: Should I stick to this approach even if some students are visibly upset over not having comprehended a lesson?

A: If a child is upset, that child has been made to feel upset, probably by a reversion to the mastery approach. In the spiral approach each situation offers something for each child. The child will only be upset if your attitude indicates that there was more to the situation than the child was able to understand. Every experience in life may be understood at different levels. This is naturally accepted by people except when it is indicated to them that they should have a deeper understanding.

Rather than abandoning the spiral approach, if a child seems upset, stick more closely to it. Find a way for the child to express what understanding he or she does have and accept that expression in a positive manner.

15. Q: What provisions are made for the slow learner?

A: The spiral approach provides for the slow learner by introducing situations that each child can experience and learn from at his or her own level. The next time the spiral reaches a similar situation each child's level of understanding will be somewhat higher but again the experience and learning take place at each child's own level. This is as true for the slow learner as it is for any other student.

The slow learner should participate in math with the other students in the class but might benefit from additional small group work. The additional time is not to rehash or drill on the situations presented in class, but rather to provide additional experiences for these students. See the book "Mathematical Play Therapy" for suggestions for working with slow learners.

16. Q: While providing for the slow learner, how do I challenge the brighter students?

A: The spiral approach is very good for addressing the needs of all levels of students and gives students a chance to excel in mathematics. The situations presented are often open-ended and allow the student to experience and understand them at as high a level as the child is capable.

In order to accommodate the various ability levels in a given class, the worksheets generally are written at three levels of difficulty denoted by stars. "*" represents an easy worksheet, "***" represents an average level worksheet, and "****" represents a difficult level worksheet. All students should begin with the "*" level worksheets for a given lesson or sequence of lessons and proceed as far as possible through the other two levels. In classes containing the usual range of abilities, most of the students should succeed with the one-star worksheets; the two-star worksheets should be appropriate for about two-thirds of the class; and approximately one-third of the students will find the three-star worksheets challenging and enjoyable to do. The workbook series are usually written at three different levels of difficulty in the same manner as the worksheets.

If your school is flexible, there is also the possibility of letting your students take math with a higher or lower grade level class. For example, your very bright second graders could go to a third or fourth grade class for math instruction.

17. Q: Is it necessary to follow the suggested schedule exactly?

A: This program is not meant to be a static sequence of exercises to be followed rigidly. The lessons in this guide are suggestions and the time allotments are merely approximations based on our

own experience teaching the materials. If this will be the first time you are teaching the program and you would like specific suggestions for organizing the lessons, we offer some guidelines and a suggested schedule of lessons. You may wish to follow it at the beginning of the school year and then deviate from it as the needs of your students may dictate.

It is probably best to follow a one-day, one-strand pattern when you organize your own schedule, rather than to spend two or three full weeks "covering" one particular topic. Interchange the strands and the days of the week as you please. Three days each week should be devoted to the two parts of the World of Numbers strand and the Workbook strand.

Exercise your own professional judgement in deciding when to stop one particular lesson and when to take it up again. Just be sure that you do not insist that every single child understands every single aspect of a lesson before you go on. We suggest that you make a note of where you end a given lesson so that you will know where to begin the next time you teach it.

18. Q: How do I handle individual students who are new to CSMP?

A: In most cases, the differences between new and veteran students tend to fade rather quickly. One exception might be in lessons involving the Minicomputer. Veteran students are usually adept with the Minicomputer and often are very good tutors for the new students in the use of this abacus. In addition, the storybook "Two By Two" is suggested as an aid for new students who are unfamiliar with the Minicomputer. Some of the storybooks and story-workbooks written for younger children can be used to introduce new students to the languages of strings and arrows.

19. Q: How do I handle whole classes who are new to CSMP?

A: CSMP does not assume that Kindergarten and first grade students have any prior experience with CSMP. The Kindergarten and First Grade Programs will need no adaptation for classes who are new to CSMP.

Second grade classes which are new to CSMP will need to be introduced to the CSMP languages. A suggested adaptation of the second grade program is included in the CSMP Mathematics for the Upper Primary Grades (UPG) Part I teacher's guide.

Third grade classes which are new to CSMP will need to become fluent in the CSMP languages before beginning the lessons in UPG Part II. A special schedule has been prepared for these classes which consists of prerequisite lessons and workbooks from Part I followed by a modified Part II program.

Fourth grade classes which are new to CSMP will begin with CSMP Mathematics for the Intermediate Grades, Entry Module I which lasts one semester, before the class begins the regular fourth grade program. Future development plans provide for the creation of Entry Modules II and III for entry into the CSMP curriculum at the fifth and sixth grades, respectively.

20. Q: What should I do with my class if they did not finish the previous year's schedule?

A: No special measures are required. Each level of the CSMP curriculum begins with introductory lessons. In addition, the spiral approach to the lessons permits each student to participate at his or her own level of understanding, even if that level is not quite as high as it might have been if the previous year's schedule had been completed.

21. Q: How do I handle a student who has an extended illness?

A: The spiral approach used by CSMP permits a student to miss one or two weeks of school without needing much catch up work. When the student returns to school either you or one of his or her classmates can assist him or her with any material for which some help is deemed necessary.

If a student misses several months of classes and experiences difficulty with any of the CSMP languages upon return, he or she can be given the same help as you would give a transfer student new to CSMP.

Because of the nature of the materials and the parent's possible unfamiliarity with the languages, it is probably best not to have the worksheets and/or workbooks sent home to a sick child. In fact, without the lesson or collective introduction which often precedes the individual work on worksheets and workbooks, it would be practically impossible for the student to proceed at all.

22. Q: How can I assess student progress and assign grades?

A: CSMP does not believe that it is either desirable or reasonable for teachers to attempt to evaluate student mastery of skills from day to day or even from week to week. On the other hand, teachers, parents and even the children themselves, benefit by some monitoring procedure that records involvement in activities and assesses progress through the program.

Most teachers have their own monitoring devices with which they are comfortable. It is recommended that such devices be continued.

23. Q: How do CSMP students do on standardized tests?

A: While CSMP is opposed to the philosophy underlying standardized testing, test results are available to us from several sources. The results indicate that CSMP classes generally do as well or better on tests than do non-CSMP classes. It should be pointed out that standardized testing and traditional programs are correlated and that the CSMP student has been exposed to a great deal of mathematics that is not taught in more traditional programs or considered in the construction of standardized tests. There are also many immeasurable or unmeasured areas such as the enjoyment of mathematics, strategic thinking, organizational ability and depth of understanding of the methods, power and limits of mathematics where we feel CSMP students have a definite advantage.

24. Q: What problems would CSMP students have, if they return to a more traditional program in the next school year?

A: There should be no problems for CSMP students who return to a traditional program. Test results indicate that CSMP students are at or above the level of non-CSMP students in material covered in traditional courses. In addition, CSMP students have encountered many situations that are not introduced in traditional programs nor tested for on standardized tests. (See Question 23.) In particular, their reasoning ability should be much sharper than their non-CSMP counterparts.

There are, of course, differences in timing as to when different programs introduce the standard algorithms and often CSMP's introduction will occur later than in some of the other programs. The reason for this is our belief that it is essential that the creative freedom of the student not be impaired by a premature emphasis on the standard algorithms for adding, subtracting, multiplying and dividing. We believe that many of the present computational difficulties experienced by elementary and junior high school students

were caused by their spending an inordinate amount of time trying to master the algorithms too early in their mathematical education. This shuts the students off from other experiences necessary for their mathematical growth. Students need plenty of time to explore the world of numbers. Mastering an algorithm should be the last step of this exploration, a sort of synthesis of previous experience. Furthermore, as electronic hand-calculators become available to nearly everyone, algorithmic mastery as a prerequisite to other numerical experiences should become less important.

CSMP students can usually solve a problem before having learned the standard algorithm. If they enter a program that expects an algorithm they have not learned, they are still more than adequately prepared for the algorithm and should have no trouble quickly learning to use it.

CSMP introduces the usual addition algorithm in the second grade, one of the two standard subtraction algorithms in the third grade, the multiplication algorithm in the fourth grade, and the division algorithm in the fifth grade.

25. Q: Why doesn't each student receive every workbook and storybook?

A: The students are of various ability levels in every class. Some students will be able to work only on "*" and others on "*" and "***" but not on "****" problems, so there is no need to give a complete set of workbooks to every student. (See Question 16.) For this reason the usual assortment for a class of 30 students contains 30 of each "*" workbook (one for each student), 20 copies of "***" workbooks, and 10 copies of the "****" workbooks. If we were to include one copy of each workbook for every student, many would be wasted and the cost of the program would increase.

You will find enough storybooks included to provide one for each pair of students. While this is adequate, the ideal situation is

for each student to have his or her own copy of each book and to be allowed to keep it when the lesson is over. Additional copies may be ordered as individual items on the order forms.

26. Q: How can I explain CSMP to parents?

A: You will find a letter to parents at the end of this section which may be reproduced and sent home with the students.

The best way would be for your coordinator to organize a workshop for interested parents. The parents can then see the languages of the program in use and have some of the same experiences their children have had. Workshops can be of almost any length but anything short of two hours does not allow for much familiarity with the material. In such a workshop we would suggest that the language of the Minicomputer be explained with particular emphasis on its use in presenting situations where mental arithmetic is called upon and in the development of algorithms (for example, physically carrying a checker from the ones' to the tens' board in the " $8 + 2 = 10$ " play, paralleling carrying in the paper and pencil operation). A lesson using the storybook "Where's My Nose?" provides a good example of the power of the language of strings. A nonnumerical example, such as the brother-sister graph found at various grade levels, and a numerical example, such as building a road from 5 to 55 using $+3$ and $5 \times$ arrows, will provide exposure to the language of arrows.

27. Q: What are some ways to organize materials and manage my class?

A: It is very important to organize your CSMP materials before school begins. Decide where you will store the materials once they are organized. Filing drawers, shelves, boxes, or closets are all satisfactory as long as you can easily get to the materials. Take them all out of their shipping boxes and spread them on several tables and/or desk tops in your classroom so they can be organized into the order you will be using them.

Workbooks: The teacher's guide (or guides) are needed from second grade onward to find the order in which the workbooks are used. Find the table of contents in the Workbook strand. Collect all the workbooks in the order which they are listed. Some workbooks have identical titles with only the number of stars in the upper right-hand corner differing. Always gather the one-star level workbooks first, then the two-star workbooks, and finally the three-star workbooks of a particular title. In some cases there are only two levels (i.e., no three-star workbooks exist) for a given series and in other cases only one level. In these cases, ignore the statement above regarding three-star and/or two-star workbooks respectively.

After organizing all the workbooks, place them in an upright position in your file drawer, box, shelf, etc., so that the first workbook to be used is at the front of the stack.

Worksheets: This grade level has a separate worksheet booklet for each student and you should decide whether you will store these books and pass them out when a particular lesson calls for a worksheet, or simply let the students keep their own worksheet books in their desks.

Each worksheet is to be removed from the booklet before the student begins working on it. Once a worksheet is removed from the booklet it cannot be replaced, so make sure students are careful about tearing off the correct sheet.

Student Manipulatives: Storage of each manipulative in a box, properly marked, is the best way to make everything accessible: a shoe box for the compasses, an envelope for the spinners, a shoe box for marbles, and so on.

Individual Minicomputer sheets can be better preserved if you use a clear contact paper over the top. This will prevent bending, etc.,

of the sheets.

Additional Materials: Be sure that you have plenty of high quality colored chalk, unlined white paper, and that your students have colored pencils, pens, or crayons. Plan well ahead to be sure you have any needed materials that are not supplied by CSMP.

Keep all teacher materials where they are easily accessible. You will find the guides for grades K - 3 easier to use if they are put in looseleaf binders. The demonstration Minicomputer boards and the number line should always be on display in the room. You may wish to glue the Minicomputer boards on pieces of cardboard to keep the edges from curling.

28: Q: Much of the CSMP material is new to me. How do I learn enough to teach a class?

A: Every CSMP teacher should have been trained by his or her coordinator. Kindergarten teachers receive eight hours training, first grade teachers sixteen hours, second grade teachers twenty-four hours, third grade teachers thirty-two hours, and fourth grade teachers forty-hours. During the training sessions the CSMP content is taught to new teachers and suggestions are given for organization of their classes.

If the suggested training schedule has been followed, many lessons taken from the CSMP curriculum, or very similar lessons, have been presented. The schedule also makes use of the spiral approach so that training sessions aim not only to teach CSMP mathematics but to familiarize teachers with the curriculum and pedagogy of CSMP.

29. Q: What should be done when there is a substitute teacher?

A: Some school systems have included in their CSMP training program people who frequently substitute. When this is not the case, it would be very difficult for the untrained substitute to teach from CSMP materials. One solution is to arrange for another CSMP teacher to teach math to your class while the substitute teacher teaches another subject to that teacher's class. If that can't be arranged and you will only miss one or two days, you may find certain CSMP material, uncompleted workbooks for example, that the substitute can manage. Otherwise, use those days to cover material not included in the CSMP curriculum, such as telling time. If your absence will be an extended one, arrangements should be made with your coordinator to train a substitute, rearrange schedules to put a CSMP teacher in your class, or otherwise provide for your students.

30. Q: The program makes use of the student's creative freedom. How much room is there for the teacher's ingenuity?

A: CSMP not only allows for your ingenuity but depends on it. While the lessons may appear to be very rigid by presenting a dialogue for teacher and student, it should be remembered that these are suggested questions, answers, and comments based on our experience with development classes. Even if you were to follow your part of the "script" your students are likely to offer responses which differ from those we have listed. It is the student's response that will determine the content and direction of your next question or comment.

In a traditional program your creativity is called upon the first time you develop a particular lesson, and to a lesser extent, as you revise lessons in succeeding years. There is little room for spur-of-the-moment creativity once a lesson begins. The CSMP curriculum, by creating a free and open situation emphasizing the student's creative freedom, continually presents you with unanticipated student responses and calls on your ingenuity at all times.

TO THE PARENTS OF CSMP STUDENTS

Your child is participating in the Comprehensive School Mathematics Program's (CSMP) elementary school curriculum, developed under a contract with the National Institute of Education. While the program cannot be fully explained in so brief a format, it is hoped that this letter will give you some feeling for the substance of your child's mathematics education.

The content goals of the CSMP K-6 program include a development of the standard number systems and their operations (including an early introduction to the integers, rational numbers and decimal numbers) interwoven with a two-pronged development of geometry (a rich variety of experiences in combinatorial and metric geometry together with a carefully developed sequence of activities in parallelogram (affine) geometry). On an equal footing with the above, the CSMP program includes a strong component of probability, statistics, and combinatorics which extends throughout grades 1-6. Powerful pedagogical devices are exploited providing several non-verbal languages which render the unifying mathematical ideas underlying the content accessible to very young children. Through these languages, children are able to apply mathematical ideas effectively and creatively to explore the content directly, unencumbered by the terminology and notation which usually pervade the early stages of the curriculum and stand as a barrier between the children and the content. These languages include:

- 1) The Language of Strings. This "language" mimics the basic methods of collecting and classifying data relevant to the problems at hand. That is, it deals with the fundamentally useful and important mathematical notion of set.
- 2) The Language of Arrows. This graphical language models the fundamental process involved in comparing and analyzing sets and operations on them. That is, it deals with the mathematical notions of relations and functions including, but not restricted to, ordering relations and

the numerical processes of adding to, subtracting from, multiplying by, and sharing equally among.

- 3) The Language of the Papy Minicomputer. The Papy Minicomputer, an extremely effective paper abacus invented by Georges Papy, models the positional structure of our system of numeration and lends itself to a multitude of numerical algorithms, including the standard algorithms for the basic numerical operations.

Among other things, these languages allow for the development of the basic operations on numbers in a natural way based on previous experiences with the languages. They also provide a context in which situations arise that call for the repeated use of arithmetic operations without resorting to drill exercises, which most children find boring.

Another tool not unique to, but used by CSMP, is the hand-calculator. The hand-calculator allows the child to study problems that call for calculation beyond the child's ability or in which the calculation is so extensive as to cause the child to lose interest. The calculator is also used in a way similar to the use of languages mentioned above. Properly used, it can help lead to the development of important mathematical ideas and present situations for the interesting use of mental or paper and pencil arithmetic.

The content is learned in an atmosphere of constant applications utilizing a "pedagogy of situations."^{*} These vary from simple story situations in the early primary grades to challenging applications to nontrivial simulations of real world problems in the upper grades.

One of the tenets of CSMP's development philosophy is that mathematics is a unified whole and should be learned as such. As a result, along with CSMP's concern with the variation in ability levels in the typical heterogeneous class, the curriculum is structured in a spiral format throughout the K-6 program. The content is completely sequenced in spiral form so

^{*} Inspired by the work of the mathematician and psychologist, Caleb Gattegno.

that each student is brought into contact with each area of content continuously throughout the program. This consciously precludes atomizing the content and mastering each bit before continuing to the next. Rather, each student spirals through repeated exposures to the content, building interlocking experiences of increasing sophistication.

Another tenet of CSMP's development philosophy, based on years of experimentation in the classroom, is that no single method of classroom management suffices to meet the needs of every student. Hence, the program is constructed to allow numerous opportunities for whole class participation, small group participation, and individual independent experiences. Individualization throughout the K-6 program is achieved through graded series of workbooks integrated into the basic sequence of lessons. Due to the spiral nature of the curriculum, each student can engage in a workbook series without losing touch with the progress of the class while at the same time working his or her way up through the series to the limits of his or her ability.

A feature unique to the CSMP program is a collection of storybooks written by Frederique Papy (STORIES BY FREDERIQUE) and a collection of storyworkbooks (THE CSMP LIBRARY). These probe the affective domain of learning and bring children into contact with mathematics at levels until now unexplored.

CSMP does not have its own system of student assessment and progress reporting. While standardized testing indicates that CSMP students develop the "basic skills" as early or earlier than non-CSMP students, we hope you will not be obsessed with your child's development of these "skills". Each child is an individual and should not be judged against norms but on what he or she can and does accomplish. Many extremely important areas --- development of self-confidence, ability to think and reason, etc. --- are very difficult to measure and grade. If you have any questions regarding the progress of your child, his or her teacher is in the best position to answer those questions.

THE CSMP STAFF

Workbooks

Comprehensive School Mathematics Program

**Mathematics
for the
Intermediate Grades
Part I**

Workbooks

CEMREL, Inc.

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WORKBOOKS

INTRODUCTION

There are many opportunities for the children to work individually during the course of the lessons described in the four content strands. In this workbook strand, however, it is this individualized work which becomes the chief end of the lessons. It has been our goal in this strand to provide the students with opportunities to

- review many of the ideas they have met in the content strands
- apply their acquired knowledge to new situations requiring various kinds of strategic thinking
- learn how to read and use mathematics workbooks and story-workbooks.

The following workbooks are provided:

Collection of Problems 1

Collection of Problems 2

The following story-workbooks are provided:

A Book About Me

Halloween Puzzles

Not Too Close

Summer Camp

The two workbook series, Collection of Problems, 1 and 2, are written at three levels of difficulty. These levels of difficulty are coded as follows: easy - *; average difficulty - **; very challenging - ***.*

We suggest that you devote one day a week to this strand. (For definiteness, we have assumed you have chosen Wednesdays.) We provide a com-

*The workbook Tim and I replaces the *** level in Collection of Problems 1.

plete suggested schedule for Wednesday's lessons immediately following this introduction. It is suggested that you follow this schedule more or less as given, even if many of your students have not finished a particular workbook after the two or three days that are allotted to it. As a general rule, students at this age do not enjoy remaining on the same topic for too long. And besides, it is a basic principle of the CSMP spiral approach that a variety of experiences is preferable to any attempt to master a given topic by continued "plugging away" at it. This principle applies as much (if not more so) to the workbook strand as it does elsewhere in the curriculum. If you do feel, however, that more time is needed for a particular workbook, you may find it convenient to continue work on that workbook during one (or more) of the free days that occur in the suggested schedules for the four content strands.

For each of the story-workbooks dealt with in this part of the Teacher's Guide there are collective lessons that accompany the story-workbook. These are followed by an answer key for that story-workbook. There is also an answer key for the two workbook series but no introductory collective lessons. These answer keys are presented in reduced format, each page actually containing reduced copies of four of the pages of the original booklet, with solutions written in.

For each title we suggest that all students start work at the easiest level of difficulty and then proceed as far as they can through the other levels in the time that is to be devoted to that title. You can expect that, under normal circumstances, about two-thirds of the students will work through the second level of difficulty, and about one-third will reach the most challenging level. These assumptions are reflected in the fact that a full classroom set of materials contains only 20 copies of each of the ** workbooks and only 10 copies of the *** workbooks. Additional copies of these workbooks (all levels) can be ordered as separate items on the order form for Part I of the Intermediate Grades Program.

The workbooks provide an excellent instrument with which you can assess the progress of your students on a week-by-week basis. To use this instrument effectively certainly does not require that you check every page

and every problem for each student. Each of you should develop the sort of procedure for checking students' work with which you can feel most comfortable. This might take the form of checking one or more specific pages, discussing some particular mistakes with individual students and letting them correct their work, or simply looking carefully at the first few pages to be sure the students have understood the general idea of the problems in a particular book. In our experience, an effective procedure is to clip to the front cover of each checked workbook a strip of paper on which can be indicated your assessment of the student's work on each page you check. For this purpose, you might use the following symbols:



meaning "well done" or "correct"



meaning "something wrong, try again"



meaning "did you forget this problem"
or "do you need a hint?"

On the next page we show the front cover of one student's Collection of Problems 1 (*) workbook after it has been checked.

Immediately following the suggested schedule for Wednesday's lessons we provide a progress chart to help you keep track of

- where each of your students is on any particular workbook day
- how many levels each student has completed under each title
- your personal degree of satisfaction with the student's work on each title.

Name Carmen S.

*

2 

3 

4 

5 

6 

7 

8 

9 

10 

11 _____

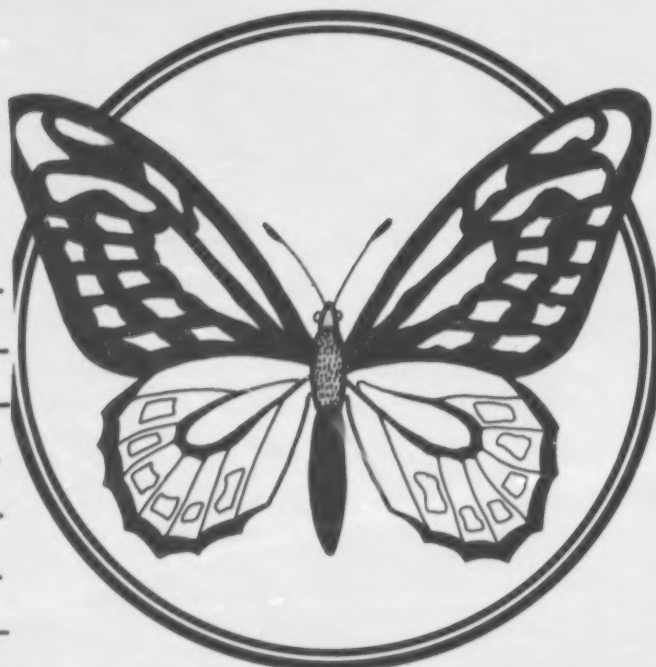
12 _____

13 _____

14 _____

15 _____

16 _____



COLLECTION OF PROBLEMS # I



(Note: this is the way the paper was returned to Carmen at the beginning of the second lesson devoted to this workbook series. The blanks on lines 11-16 indicate that she had not yet worked on these problems. Before Carmen begins on these pages during the second lesson, she should correct her mistakes on page 7 and finish her work on pages 4 and 9.)

In particular you should be on the watch to see if for any of your students even the easiest workbooks are too difficult. Such students will probably benefit a great deal if you were able to devote some time with them to using some of the suggestions made in the book "Mathematical Play Therapy".

We now provide a suggested schedule for Wednesday's lessons. The placement of the various workbooks has been determined by bearing in mind what knowledge is assumed and where the lessons that deal with that information occur in the suggested schedule for the four content strands. Hence you are warned to be particularly careful when considering possible modification of this suggested schedule. Whatever modification you might like to make, we strongly suggest that you use the same day each week for the workbooks.

Here is a list of some of the important points to bear in mind when working in this strand:

- All students begin at the easiest level of each title^{*} and progress as far as they can.
- All students begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students are expected to complete all levels of a given title. Only the faster students will reach the most challenging level. The slower students may succeed only at the easiest level, although you should not assume that this will automatically be the case—surprises are not at all uncommon in this area.
- Read the introductory material for each workbook title and give the short introductory collective lesson(s).

^{*}The Tim and I workbook is considered part of the Collection of Problems 1 series, so starting work on it does not constitute beginning a new title.

SUGGESTED SCHEDULE FOR WEDNESDAY'S LESSONS

WEEK	WEDNESDAYS	WEEK	WEDNESDAYS
1	A Book About Me	10	Summer Camp
2	A Book About Me	11	Summer Camp
3	A Book About Me	12	Collection of Problems 2
4	Collection of Problems 1 Tim and I	13	Collection of Problems 2
5	Collection of Problems 1 Tim and I	14	Collection of Problems 2
6	Collection of Problems 1 Tim and I	15	Not Too Close
7	Halloween Puzzles	16	Not Too Close
8	Halloween Puzzles	17	Not Too Close
9	Halloween Puzzles	18	FREE DAY

Worksheet Progress Chart

[illegible]

**A BOOK
ABOUT ME**

A BOOK ABOUT ME

Lesson 1

Tell your students that a secret number will give them some clues to help them find who it is. Write the first clue on the board:

I am an odd number.

T: Could the secret number be 19? (Yes); $19 + 4$? (Yes); $19 + 5$? (No); $19 + 10$? (Yes); $19 + 13$? (No); $19 + 100$? (Yes); and so on.

Emphasize the fact that

Odd + Odd = Even

Odd + Even = Odd

T: Could the secret number be 13? (Yes); 2×13 ? (No); 3×13 ? (Yes); 4×13 ? (No); 5×13 ? (Yes); 10×13 ? (No)

Ask the students to show on the Minicomputer some numbers that it could be and some numbers that it couldn't be. Have them notice that each time there is no checker on the 1 - square, the number is even. Then write the second clue on the board:

I am an odd number.
I am smaller than 100.

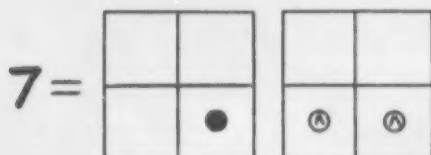
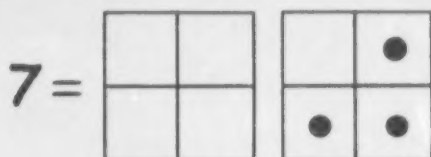
T: Could the secret number be 43? (Yes); 2×43 ? (No, because this number is even); 3×43 ? (No, because this number is larger than 100); 4×43 ? (No, because this number is even . . . and larger than 100)

Write the third clue on the board:

I am an odd number.
 I am smaller than 100.
 I can be put on the Minicomputer with exactly three checkers (positive or negative).

T: Could the secret number be 7? (Yes)

Have the students show 7 with three checkers on the Minicomputer.
 For instance,

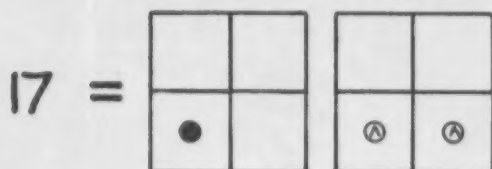


Don't ask for all the solutions, but encourage the students to give more than one answer.

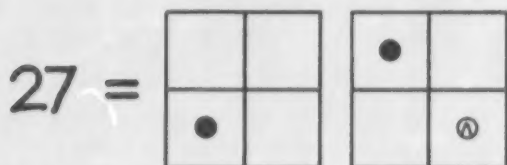


2

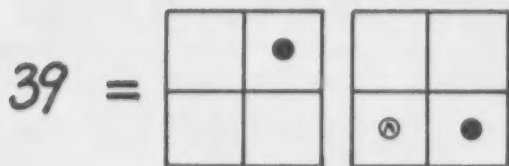
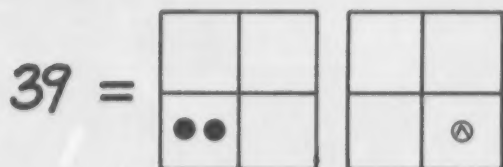
T: Could the secret number be 17? (Yes)



T: Could the secret number be 27? (Yes)



T: Could the secret number be 39? (Yes)

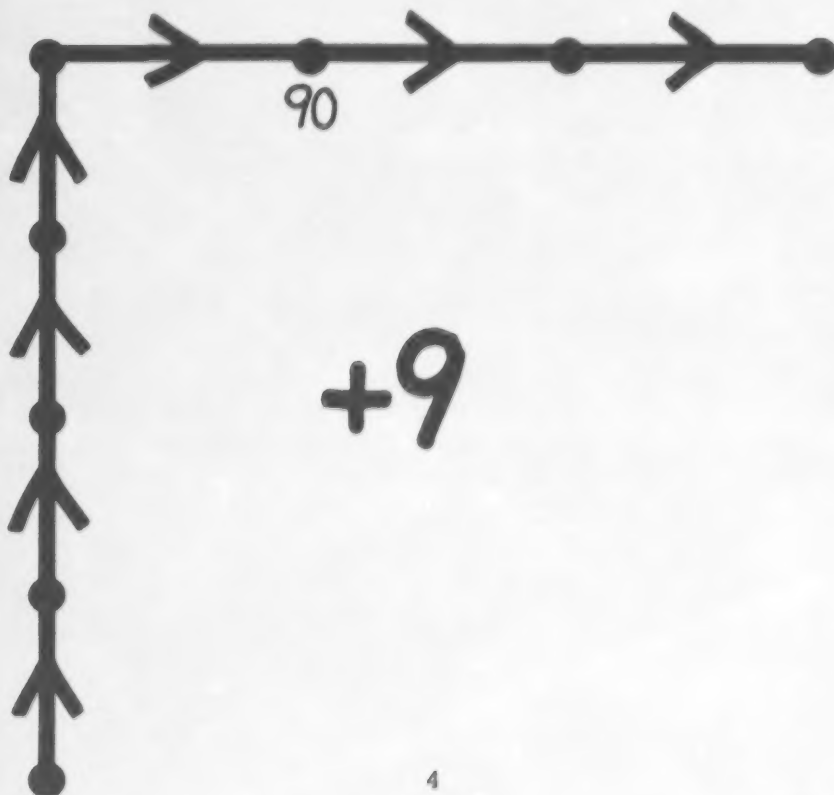


$$39 = \begin{array}{|c|c|} \hline \bullet & \oplus \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \oplus \\ \hline \end{array}$$

T: Could the secret number be 77? (Yes)

$$77 = \begin{array}{|c|c|} \hline \bullet & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \oplus \\ \hline & \bullet \\ \hline \end{array}$$

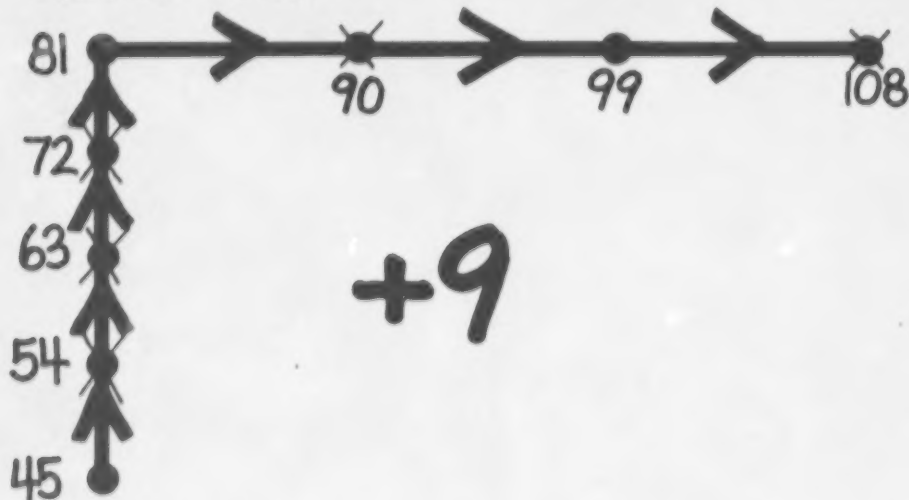
Draw this picture and the fourth clue on the board:



I am an odd number.
 I am smaller than 100.
 I can be put on the Minicomputer
 with exactly three checkers (positive
 or negative).
 I am in this arrow picture.

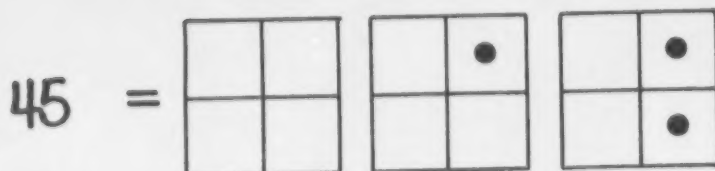
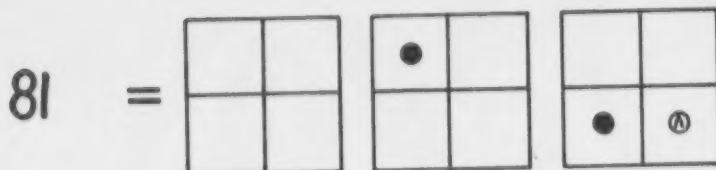
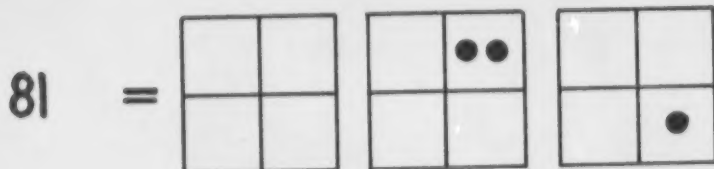
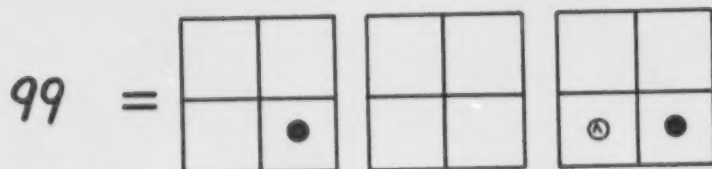
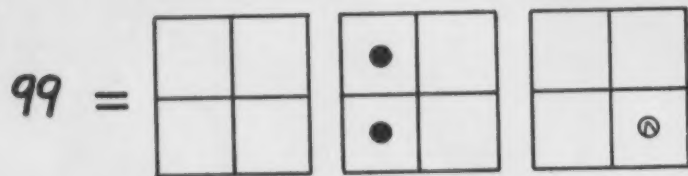
Have the students copy the arrow picture on a sheet of paper ; have them label the dots and cross out those that the secret number could not be.

After a while , let some students complete the picture on the board and explain their answers:



54, 72, 90 and 108 are crossed out because they are even (and the last one is not smaller than 100); 63 is crossed out because there is no way to put it on the Minicomputer with three checkers.

Have the three other numbers shown on the Minicomputer with three checkers:



T: Now you know that the secret number is 45 or 81 or 99.

Write the last clue on the board:

I am a square number.

Explain that square numbers are whole numbers that can be obtained from calculating

$$\square \times \square$$

where the same whole number is put into each frame.

Let the students give some examples of square numbers ($2 \times 2 = 4$; $3 \times 3 = 9$; $5 \times 5 = 25$; and so on).

T: Write on a sheet of paper the name of the secret number.

Answer:

81

Give a copy of "A Book About Me" to each student and let them work by themselves for the rest of the time.

Don't forget to collect the workbooks at the end of the lesson.

Lesson 2

Return their copy of "A Book About Me" to all students and have a collective checking of the problems on pages 1 and 2. Then have the students work on their own for a while.

Fifteen minutes before the end of the lesson draw the picture from page 4 on the board and have some (slow) students label the dots.

Ask students to cross out dots that the secret number could not be and have them explain why.

T: What do you conclude?

S: The secret number must be 24 or 28 or 40 or 44 or 48.

T: What is the next clue? Read page 5.

S: Its double is larger than 50.

T: Could the secret number be 24?

S: No, because $2 \times 24 = 48$ and 48 is not larger than 50.

T: Could the secret number be 28?

S: Yes, because $2 \times 28 = 56$ and 56 is larger than 50.

At this point your students will probably notice that the double of 40, the double of 44 and the double of 48 are obviously larger than 50. Therefore, the secret number must be 28 or 40 or 44 or 48.

Draw the picture from page 6 on the board and ask:

T: Did you already label all these dots? What do you notice about them?

S: Their last digit on the right has to be 8. Therefore, the secret number cannot be 40 or 44. The secret number must be 28 or 48.

Draw the picture from page 7 on the board and ask:

T: If "me" were 28, what would be the starting number of this arrow picture?

S: 7, and 7 is smaller than 10.

T: If "me" were 48, what would be the starting number?

S: 12, and 12 is not smaller than 10.

T: What is the secret number?

S: 28

Lesson 3

NOTE: The instructions for teaching Lesson 3 of "A Book About Me" have been written with the assumption that there are calculators for the teacher and each member of the class. Since this may not be the case in every classroom, the following paragraph suggests a way of proceeding with only two calculators. This alternative procedure will allow you to include those parts of the lesson which use calculators, although it will be, of course, less effective than if each child has full use of a calculator. (If some students have their own calculators, they should be encouraged to bring them to school for these lessons.)

The teacher must have a calculator and at least one student must have one. This student should be at the front of the class and answer any of the questions relating to the calculator, when the display is exposed. Any of the children can answer questions when the display is hidden. All of them should be encouraged to think just as carefully and stay just as involved as if they had a calculator with a hidden display. The student with the calculator can check all the answers. For each part of the lesson using the calculator, choose a new student, in order to give as many of them the experience as is possible.

Exercise 1:

T: Turn the calculator on.
Slowly press $\boxed{+}$, $\boxed{3}$, $\boxed{=}$, $\boxed{=}$, $\boxed{=}$, $\boxed{=}$ and so on.
Watch the numbers that appear on the display. What do you think of them?

S: It's counting by 3.

S: They are all multiples of 3.

Let the students play freely with the function $+3$ for a short period of time.

T: Carefully listen to the instructions now.
Press: $\boxed{C}$, $\boxed{+}$, $\boxed{3}$, $\boxed{=}$, $\boxed{=}$, $\boxed{=}$, $\boxed{=}$. (Give the instructions slowly.)

What number is on the display?

S: 12

T: Press again: $\boxed{=}$, $\boxed{=}$, $\boxed{=}$.
What number is on the display?

S: 21

T: Hide the display (with your hand or a piece of paper).
Press: $\boxed{=}$, $\boxed{=}$, $\boxed{=}$.
What number should be on the display?

S: 30

T: Check it and then hide the display again.
Press: $\boxed{=}$, $\boxed{=}$, $\boxed{=}$, $\boxed{=}$.
What number should be on the display?

S: 42

T: Check it and then hide the display again.
Press $\boxed{=}$ as many times as necessary to get 60 on the display.

...

T: See if 60 is really on the display.
Hide the display again and don't touch the calculator.
Can you predict how many times you will have to press $\boxed{=}$ to get 78 on the display?
Write your answer on a sheet of paper.

Have a student explain why it's six.

T: Press six times in a row. What number is on the display?

S: 78

T: Hide the display and press as many times as necessary to get 90. Then look at the display to see if 90 is really there.

...

T: Don't touch the calculator for the moment.
If you continue pressing , will you get 100 on the display?

S: No. We will get 99 and 102.

T: If you continue pressing (don't do it now), what will be the next three numbers that end with "0" to appear on the display?
Write the answers on a sheet of paper.

S: 120; 150; 180.

T: Check it.

...

T: Everybody should have 180 on the display now.
Don't touch the calculator any more.
If you continue pressing , will 200 appear on the display?

S: No. 198 and 201 will appear.

T: What will be the first number that ends with "00" to appear on the display?

S: 300

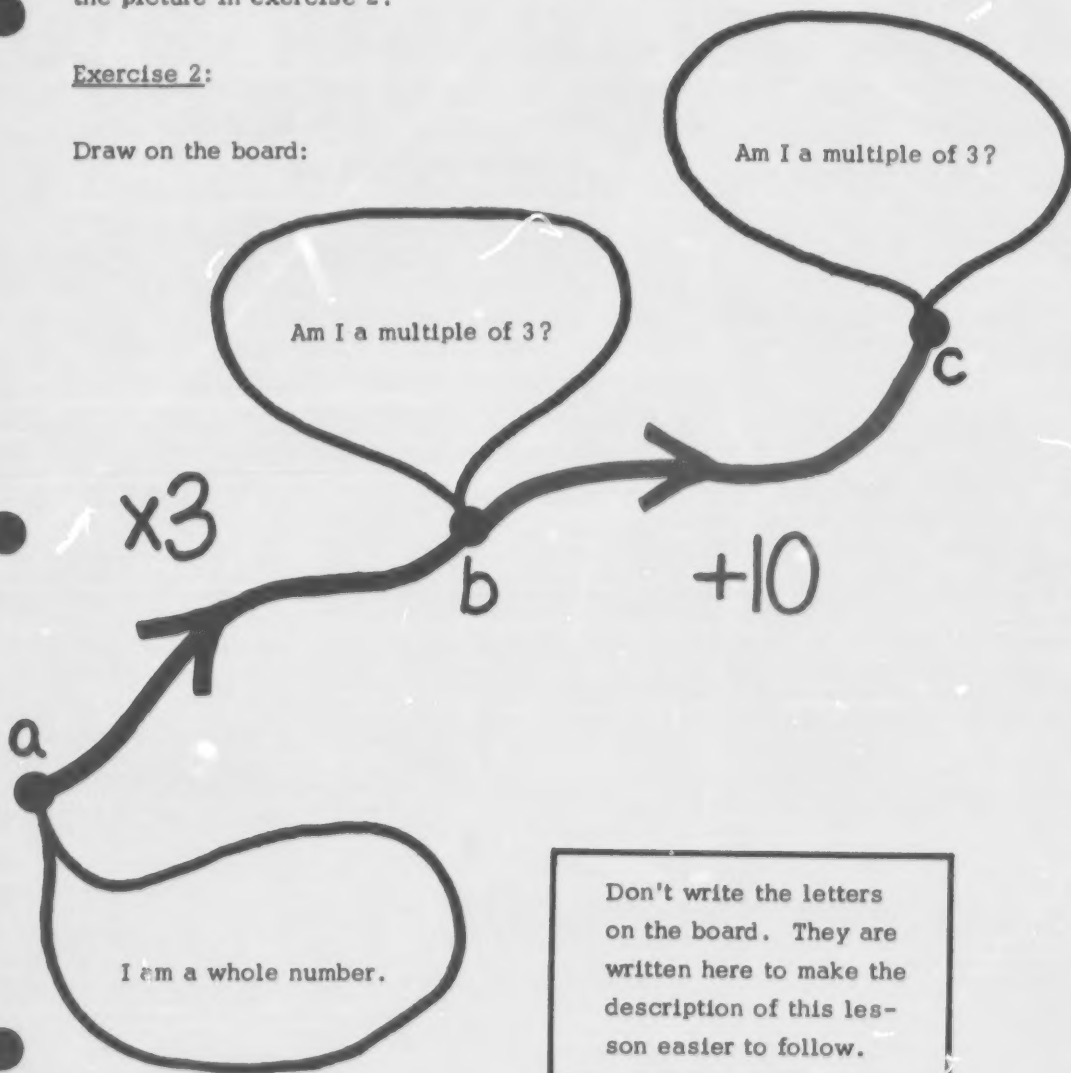
T: The next three numbers that end with "00"?

S: 600; 900; 1,200.

Let the students play freely with their hand calculators while you are drawing the picture in exercise 2.

Exercise 2:

Draw on the board:



Don't write the letters on the board. They are written here to make the description of this lesson easier to follow.

Point to a and ask:

T: What number could this dot be for?

Have the students give many examples of whole numbers.

T: What numbers are not possible for this dot?

Have students give many examples of numbers which are not whole numbers. (For example, -3 ; $\frac{1}{2}$; 1.7 ; -3.5 ; and so on.)

Point to b and ask:

T: Is this number a multiple of 3? [Answer: yes (by definition)]

Point to c and ask:

T: What about this number?

Let students whisper the answer in your ear.

T: Many of you have answered "No". Can you explain why?

S: If the first number were 1, then the second one would be 3 and the last one would be 13. But 13 is not a multiple of 3.

Emphasize the fact that

(Any multiple of 3) + 9 is a multiple of 3.

(Any multiple of 3) + 10 is not a multiple of 3.

Erase "+10" and write "-18" in blue on the board. Point to c and ask:

T: Is this number a multiple of 3?

Let several students whisper the answer in your ear and have one of them explain why the answer is "Yes".

● Erase "-18" and write "x2" in blue.

T: Is this number (point to c) a multiple of 3?

● Let several students whisper the answer in your ear and have one of them explain why the answer is "Yes". To make this point clear, ask a student to give you a multiple of 3. Let's suppose that it's 12. Write on the board:

$$12 = \square \times 3$$

Let a student write "4" in the frame. Write on the board:

$$12 = \boxed{4} \times 3$$

$$2 \times 12 = \square \times 3$$

Let a student write "8" in the frame. Do a similar exercise with one or two more examples.

● Have students work individually to complete page 8.

Exercise 3:

T: Turn your calculator on and press: $\boxed{C} \boxed{+} \boxed{5} \boxed{=} \boxed{=} \boxed{=}$
What number is on the display?

S: 15

T: Press: $\boxed{=}$, $\boxed{=}$, $\boxed{=}$. And now?

S: 30

● T: Hide the display and press: $\boxed{=}$, $\boxed{=}$, $\boxed{=}$.
What number should be on the display?

S: 45

T: Check it and then hide the display again.
Continue pressing [=] until the first number with three digits appears on the display.
What should this number be?
Write it on a sheet of paper.
Don't look at the display.

T: Many students have written "100". It's the correct answer. Look and see if 100 is really on the display. If not, try to get it.

...

T: Don't touch the calculator any more.
If you continue pressing [=] ten times in a row, what will be the last number to appear on the display?

Have the students write the answer (150) on a sheet of paper and ask one of them to explain why this number has to be 50 more than 100.

T: Check the answer with the calculator.
Put the calculator on the desk and don't touch it any more.
If you continue pressing [=] 20 times in a row, can you predict what will be the last number to appear on the display?

Have the students write the answer (250) on a sheet of paper and ask one of them to explain why the number has to be 100 more than 150.

T: Check the answer with your calculator.
Put it on your desk and don't touch it any more.
If we continue pressing [=], what will be the last number with three digits to appear on the display?
And the first number with four digits?

Have the students write the answers (995 and 1,000) on a sheet of paper.

To challenge your brighter students, you can ask a similar question with four and five digits (9,995 and 10,000). Another (optional) question could be:

T: If you continue pressing $\boxed{=}$, what will be the last number to appear on the display? (Answer: 99,999,995)

T: Is it easy to recognize any multiple of 5?

Probably the students will answer that counting by 5 is very easy. Emphasize the fact that the last digit on the right of any multiple of 5 is 0 or 5.

Have students work independently to complete page 9.

Exercise 4:

Ask the students to open "A Book About Me" to page 10 and have one of them read the clue.

T: Turn your calculator on and listen to the instructions carefully.
Press: $\boxed{C}$ $\boxed{+}$ $\boxed{3}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$. What number is on the display?

S: Nine.

T: Let's go on. Each time you see on the display a multiple of 3 which is an odd number and a multiple of 5, raise your hand silently.

Give the instructions slowly:

T: Press: $\boxed{=}$ $\boxed{=}$. . .

At this point the number on the display should be 15 and the students should raise their hands.

Continue giving the same instructions: "Press: $\boxed{=}$ $\boxed{=}$. . ."
until you get successively:

15; 45; 75; 105; 135; 165; 195 and so on.

T: What do you notice?

S: Each time, it's 30 more.

Have the students turn the calculator off and work by themselves in
"A Book About Me."

A Book About Me

1

I am an even number.

Could I be 19? no

Could I be 26? yes

Could I be 38? yes

Could I be 101? no

Could I be 108? yes

Could I be  ? no

Could I be  ? yes

Could I be  ? no

2

I am even.
I am larger than 20.

Could I be 25? no

Could I be 18? no

Could I be 200? yes

Could I be 75? no

Could I be  ? yes

Could I be  ? no

3

I am even.
I am larger than 20.
I can be put on the Minicomputer with exactly two positive checkers.

Could I be 24? yes 

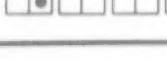
Could I be 80? yes 

Could I be 86? no 

Could I be 108? yes 

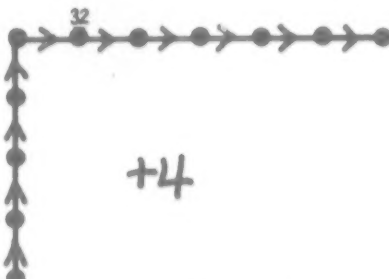
Could I be 101? no 

Could I be 34? no 

Could I be 1,088? yes 

4

I am even.
I am larger than 20.
I can be put on the Minicomputer with exactly two positive checkers.
I am one of these dots.



+4

Therefore, I must be 24 or 28 or 40
or 44 or 48.

5

You already know that I must be
24 or 28 or 40 or 44 or 48.
 My double is larger than 50.

Therefore I must be 28 or 40 or 44 or 48.

6

You already know that I must be
28 or 40 or 44 or 48.
 I am one of those dots.

+10

Therefore I must be 28 or 48.

7

You already know that I must be
28 or 48.

2x

This number is smaller than 80.

Therefore I am 28.

8

My best friend is a multiple of 3.

Could he be 20?	<u>no</u>
Could he be 15?	<u>yes</u>
Could he be $20 \div 15$?	<u>no</u>
Could he be 3×59 ?	<u>yes</u>
Could he be $47 \div 3$?	<u>yes</u>
Could he be 12?	<u>yes</u>
Could he be 120?	<u>yes</u>
Could he be 1,200?	<u>yes</u>
Could he be 121?	<u>no</u>
Could he be 122?	<u>no</u>
Could he be 123?	<u>yes</u>

My best friend is a multiple of 3
and a multiple of 5.

Could he be 25? no
 Could he be 21? no
 Could he be 30? yes
 Could he be 300? yes
 Could he be 3×5 ? yes
 Could he be $3 \times 5 \times 7$? yes
 Could he be 55? no
 Could he be 65? no
 Could he be 75? yes
 Could he be 105? yes

10

My best friend is a multiple of 3
and a multiple of 5.

He is an odd number.

He could be

15, 45, 75, 135, 225, ... and so on.

He could not be

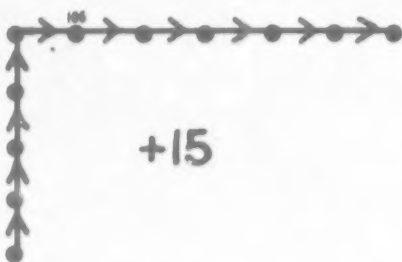
1, 2, 3, 4, 5, ... and so on.

11

My best friend is a multiple of 3
and a multiple of 5.

He is an odd number.

He is one of these dots.



Therefore he must be

45 or 75 or 105 or 135 or 165.

12

You already know that my best friend must be

45 or 75 or 105 or 135 or 165.

His double is between 100 and 200.

Therefore he must be 75 or 105 or 135.

You already know that my best friend must be

75 or 105 or 135.

His name can be written using all of these symbols, each of them only once.

$$\begin{array}{ccccc} 5 & & & & 7 \\ & \times & & + & \\) & & 8 & & (\end{array}$$

Therefore my best friend is 75.

Here is a complete list of the possible numbers which can be named according to the given clue. (If a number has more than one such name, all are given):

$$\begin{aligned} (5 \times 7) + 8 &= 43 \\ (7 \times 5) + 8 &= 43 \\ (5 \times 8) + 7 &= 47 \\ (8 \times 5) + 7 &= 47 \\ (5 + 8) \times 7 &= 91 \\ (8 + 5) \times 7 &= 91 \\ (5 \times 7) \times 8 &= 96 \\ (7 \times 5) \times 8 &= 96 \\ (7 \times 8) + 5 &= 61 \\ (8 \times 7) + 5 &= 61 \\ (7 + 8) \times 5 &= 75 \\ (8 + 7) \times 5 &= 75 \end{aligned}$$



What could this red arrow be for?
Write your answer above.

**A COLLECTION
OF
PROBLEMS #1**

(TIM AND I)

COLLECTION OF PROBLEMS #1
TIM AND I

This workbook series has three levels of difficulty (*, ** and ***).

It is suggested that all the students begin with level * and continue through as many of the workbooks as they can handle during the three lessons scheduled for this series.

The series has been designed to encourage the students to work independently; therefore no collective introductory lesson is necessary. Though there are many different types of problems, all of them will be familiar to the students because of previous work they have done in one or another of the strands.

Since the children are familiar with negative checkers, some of them may choose to use them on any of the pages where they are asked to show numbers on the Minicomputer. This is perfectly acceptable, but not necessary.

On pp. 11, 12 and 13 of level **, two or more arrows of the same color are used. It may be necessary to emphasize to your students that whenever two or more arrows of the same color occur in the same picture, they have the same meaning and must be labeled the same.

If, at any other time, you observe that many students are having difficulty with a particular page, by all means stop the independent work while you work out the problem collectively. Then let them resume their individual work as you continue to observe and give individual assistance.

A Collection of Problems #1 *

2

Write on the Minicomputer any numbers you wish, using exactly one checker.

$$\begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} - 1 \quad \begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} - 2$$

$$\begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} = 100$$

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} = 10$$

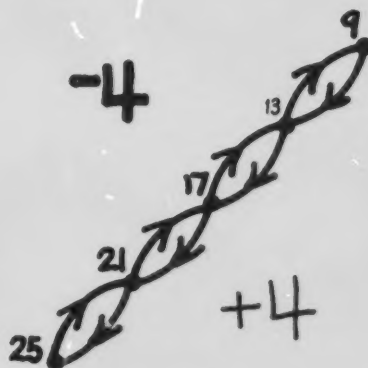
$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} = 8$$

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} = 4$$

(Many other solutions are possible.)

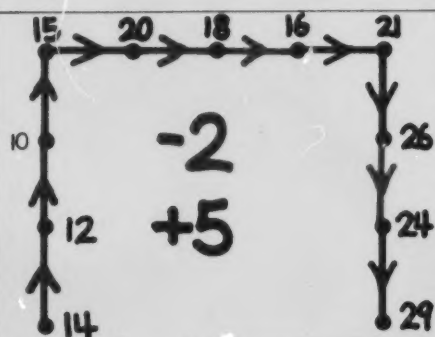
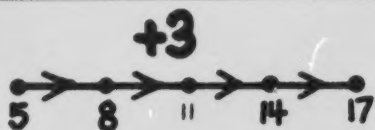
3

Label the dots. What could the blue arrows be for? Use blue to write your answer.



4

Label the dots.



5

Write any numbers you like on the Minicomputer, using exactly two checkers.

$$\begin{array}{|c|c|} \hline & \\ \hline \cdot & \cdot \\ \hline \end{array} = 2 \quad \begin{array}{|c|c|} \hline & \\ \hline \cdot & \cdot \\ \hline \end{array} = 3$$

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} = 11$$

$$\begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} = 110$$

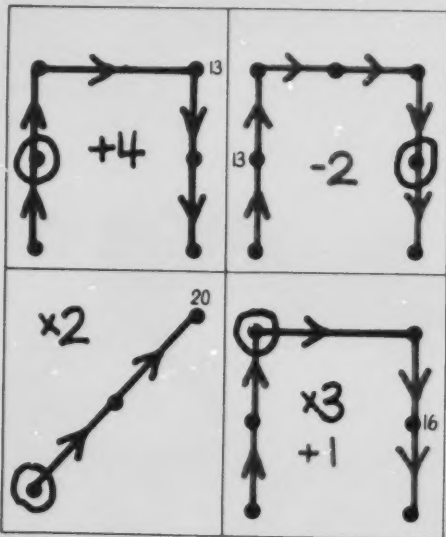
$$\begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} = 600$$

$$\begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} = 108$$

(Many other solutions are possible.)

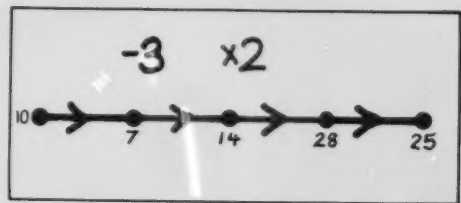
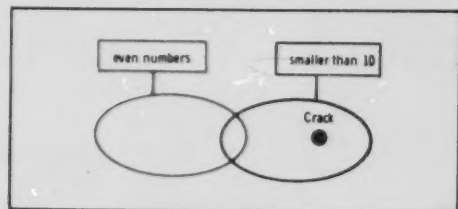
6

5 is in each of these pictures.
In each one, circle its dot.



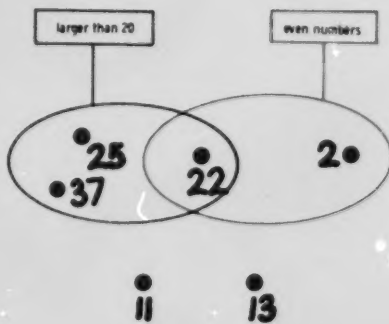
7

Crack is a secret number.
Crack is in this string picture and in this arrow picture.
Who is Crack? 7



8

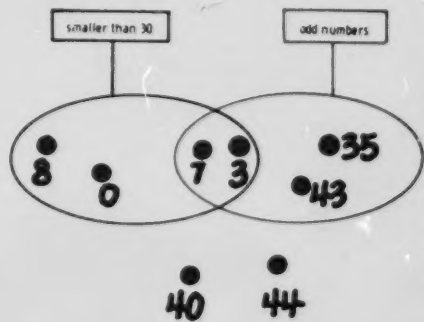
Label the dots. Many answers are possible.



(Many other solutions are possible.)

9

Draw a dot for each of these numbers:
8, 7, 35, 40, 0, 43, 44, 3



10

Write on the Minicomputer any numbers you wish, using exactly three checkers.

$$\begin{array}{|c|c|} \hline & \\ \hline \cdot & \cdot \\ \hline \end{array} - 3 \quad \begin{array}{|c|c|} \hline & \cdot \\ \hline \cdot & \cdot \\ \hline \end{array} - 7$$

$$\begin{array}{|c|c|} \hline & \\ \hline & \cdot \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \cdot \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \cdot \\ \hline \end{array} - III$$

$$\begin{array}{|c|c|} \hline \cdot & \\ \hline & \cdot \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \cdot \\ \hline \end{array} \begin{array}{|c|c|} \hline \cdot & \\ \hline & \cdot \\ \hline \end{array} - 828$$

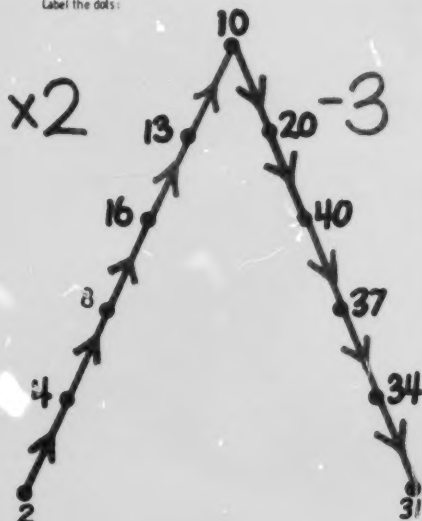
$$\begin{array}{|c|c|} \hline \cdot & \cdot \\ \hline \cdot & \cdot \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} - 1,300$$

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \\ \hline & \cdot \\ \hline \end{array} \begin{array}{|c|c|} \hline \cdot & \cdot \\ \hline \cdot & \cdot \\ \hline \end{array} - 20$$

(Many other solutions are possible.)

11

Label the dots:



12

Write these numbers on the Minicomputer:

(Many other solutions are possible.)

$$\begin{array}{|c|c|} \hline & \\ \hline & \cdot \\ \hline \end{array} \begin{array}{|c|c|} \hline & \cdot \\ \hline & \cdot \\ \hline \end{array} \begin{array}{|c|c|} \hline \cdot & \\ \hline & \\ \hline \end{array} - 34$$

$$\begin{array}{|c|c|} \hline & \\ \hline \cdot & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \cdot \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \cdot \\ \hline \cdot & \\ \hline \end{array} - 246$$

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \cdot \\ \hline \cdot & \cdot \\ \hline \end{array} \begin{array}{|c|c|} \hline & \cdot \\ \hline \cdot & \cdot \\ \hline \end{array} - 75$$

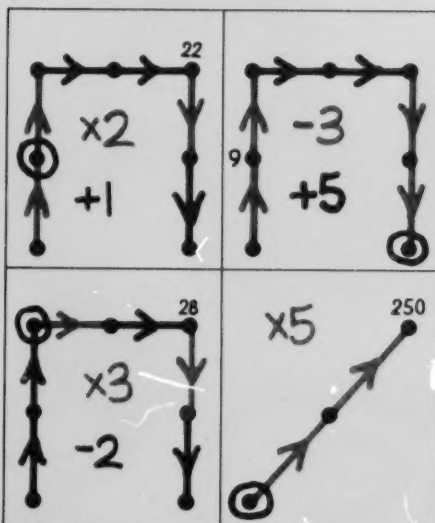
$$\begin{array}{|c|c|} \hline & \cdot \\ \hline & \\ \hline \end{array} \begin{array}{|c|c|} \hline & \cdot \\ \hline & \cdot \\ \hline \end{array} \begin{array}{|c|c|} \hline \cdot & \cdot \\ \hline & \cdot \\ \hline \end{array} - 459$$

$$\begin{array}{|c|c|} \hline \cdot & \cdot \\ \hline \cdot & \cdot \\ \hline \end{array} - 11 \quad \begin{array}{|c|c|} \hline \cdot & \cdot \\ \hline \cdot & \cdot \\ \hline \end{array} - 15$$

$$\begin{array}{|c|c|} \hline \cdot & \cdot \\ \hline \cdot & \cdot \\ \hline \end{array} - 20 \quad \begin{array}{|c|c|} \hline \cdot & \cdot \\ \hline \cdot & \cdot \\ \hline \end{array} - 24$$

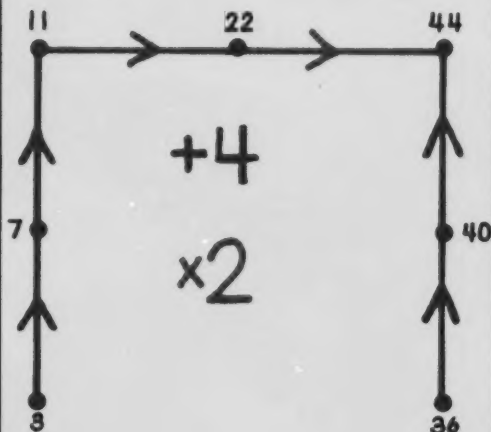
D

10 is in each of these pictures.
In each one, circle its dot.

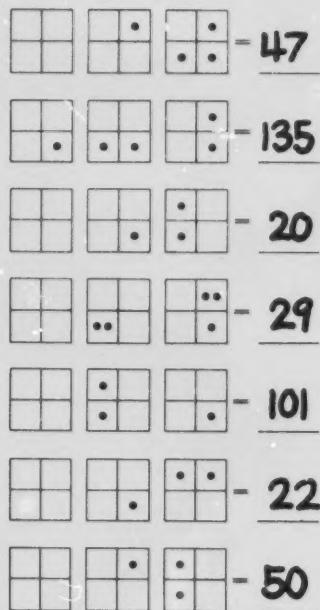


14

Flip is a secret number.
 Flip can be shown on the Minicomputer using exactly one checker.
 Flip is in this picture.
 Who is Flip? 40

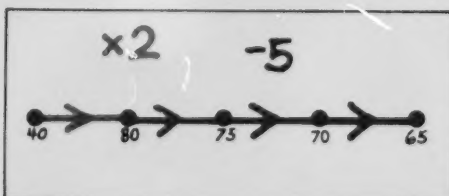
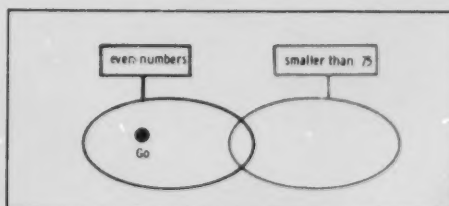


5



16

Go is a secret number.
 Go is in this string picture and in this arrow picture.
 Go can be shown on the Minicomputer using exactly one checker.
 Who is Go? 60



A Collection of Problems #1 **

2

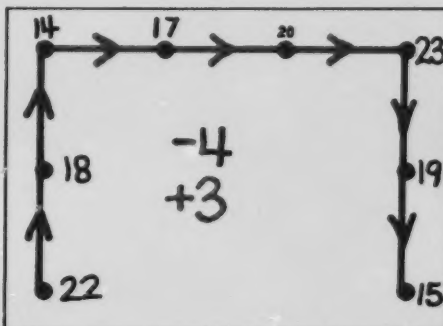
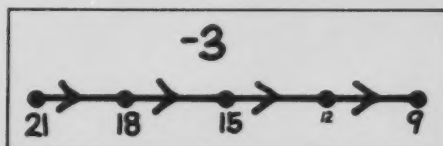
For each arrow picture, fill in the box in the upper left-hand corner with one of these tags. Then label the dots.

$\times 2$ $\times 6$ $+4$ -3



3

Label the dots:



4

(Many other solutions are possible.)

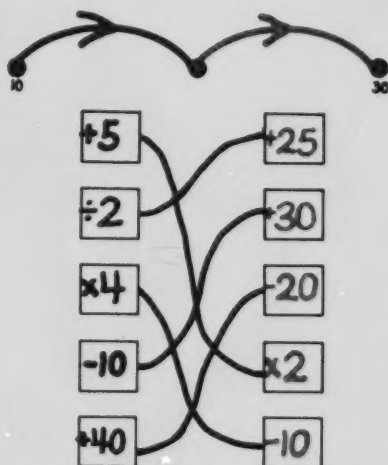
Build a blue-red road from 10 to 30.

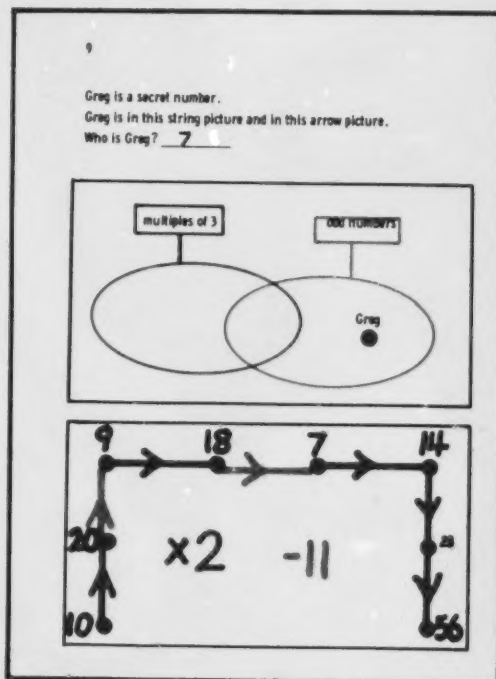
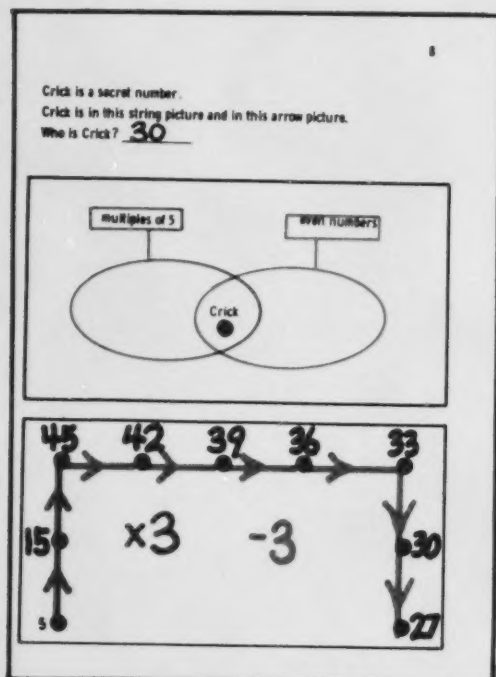
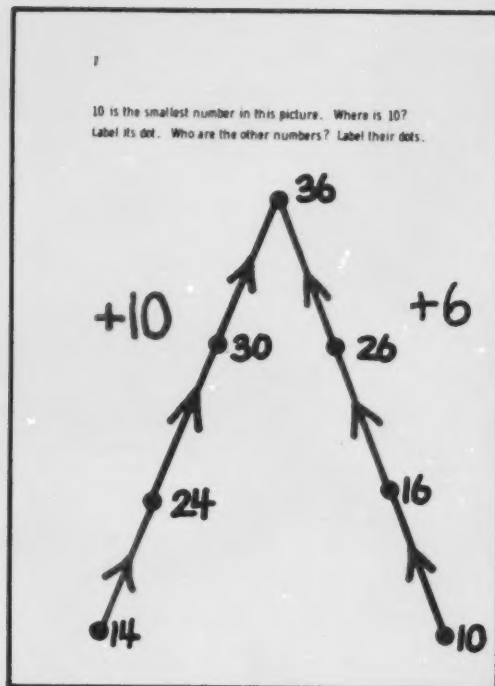
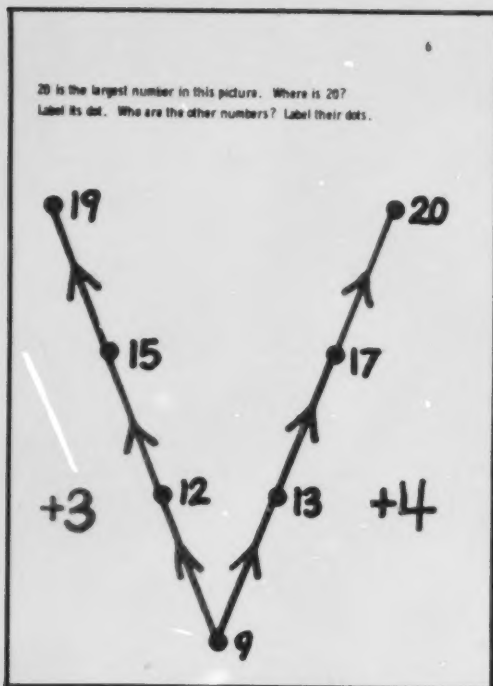
$+6$ -5



5

Pair the tags:





10

Label the dots.

-10 	$\times 2$
$\div 2$ 	-7
$\times 3$ 	$\div 3$

11

Complete the chart:

→	→
$\times 2$	$\times 4$ or $+30$
-5	$+40$
$\times 4$	$\times 2$ or $+20$
$\times 5$	$+15$
-2	$+37$
$+45$	-10

→	→
$\times 7$ or $+30$	$+5$
$\times 9$ or $+40$	-5
$\times 2$ or $+5$	$\times 4$
$\times 10$ or $+45$	-10
$\times 6$ or $+25$	$\times 2$ or $+20$
$\times 6$	$+10$

(Many other solutions are possible.)

12

In each of these pictures fill in the box in the upper left-hand corner with a label for the arrows. Then label the dots.

$+4$

The arrows can also be for $\times 10 \div 2$ or $\times 10 \div 5$, in which case the middle dot would be for $17 \div 2 = 8.5$.

$\times 2$

-12

$+32$
or $\times 4$

$+28$
or $\times 7$

13

In each of these pictures fill in the box in the upper left-hand corner with a label for the arrows. Then label the dots.

$\times 2$

The arrows can also be for $\times 10 \div 2$ or $\times 10 \div 5$, in which case the dots would be for $43 \div 2 = 21.5$ and $43 \div 5 = 8.6$.

$+16$

-21

$\times 35$
or $\times 6$

-54
or -7

14

Label the dots.
Label each black arrow using one of these tags.

-10 $\times 6$ $+10$ $\times 12$ -6 $\times 10$

15

Label the dots.
Label each black arrow using one of these tags.

$+100$ $+20$ $\times 15$ -15 $\times 100$ $\times 20$

16

Complete the chart:

$+10$	$+20$	$+30$	-20
$\div 2$	$\times 8$ or $+70$	$+40$	-40
-20	$+80$	-10	$\times 4$
$\times 2$	$\div 2$ or -80	$\div 2$ or -15	$\times 8$
$+15$	$+10$	-16	$\times 10$
-15	$\times 8$ or $+70$	$\times 2$ or $+60$	$\div 2$

Tim and I ***

If I were 10, could Tim be

40? yes
 30? no
 20? yes
 10? no
 5? yes
 2.5? yes
 2? no

If I were 100, what numbers could Tim be?

25 50 200 400

If I were -100, what numbers could Tim be?

-200 -400 -50 -25

If Tim were a positive number, could I be a negative number?

no

Could I be 2? yes

Could I be 5? yes

Could I be -5? yes

Could I be 10? no

Could I be -10? yes

What is the largest number I could be?

6

What is the smallest number I could be?

-15

6

On pages 6 and 7, write the 21 numbers that I could be.

 = -15

 = -9

 = -14

 = -8

 = -12

 = -7

 = -11

 = -6

 = -10

 = -5

7

 = -4

 = 2

 = -3

 = 3

 = -2

 = 4

 = -1

 = 5

 = 0

 = 6

 = 1

9

Write below the 16 numbers that can be shown on the Minicomputer using exactly one positive checker on the hundreds' board and one negative checker on the tens' board.

<u>90</u>	<u>80</u>	<u>60</u>	<u>20</u>
<u>190</u>	<u>180</u>	<u>160</u>	<u>120</u>
<u>390</u>	<u>380</u>	<u>360</u>	<u>320</u>
<u>790</u>	<u>780</u>	<u>760</u>	<u>720</u>

10

You don't need any more clues to find out who we are.

Is Tim a positive or a negative number? positive

(See your list of the 16 numbers on page 9.)

Am I a positive or a negative number? positive

(See your answer to the last question on page 9.)

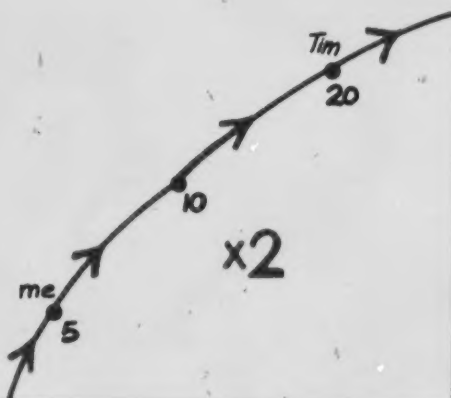
Do you remember the six positive numbers that I could be?

(See your list of the 21 numbers on pages 6-7.)

<u>1</u>	<u>2</u>	<u>3</u>
<u>4</u>	<u>5</u>	<u>6</u>

11

On this picture, write my name, Tim's number name, and the other number's name.



12



The blue arrow could be for

4x

or

+15

or

"I am smaller than you"

The blue arrow can also be for the relation "is a divisor of" as well as any other relation containing the ordered pair (5, 20). Do not be concerned if most students are not able to fill this third blank correctly. It is included to accommodate those students who enjoy being creative.

Here are some of my names. Complete them.

$$1,000 - \boxed{995}$$

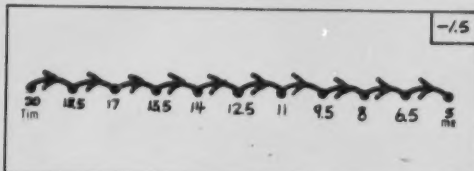
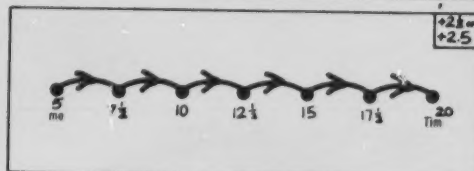
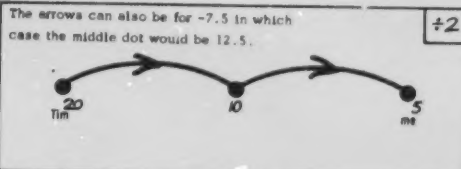
$$(2 \times \boxed{6}) - 7$$

$$(\boxed{7} + 3) \div 2$$

$$2 \times \boxed{2.5}$$

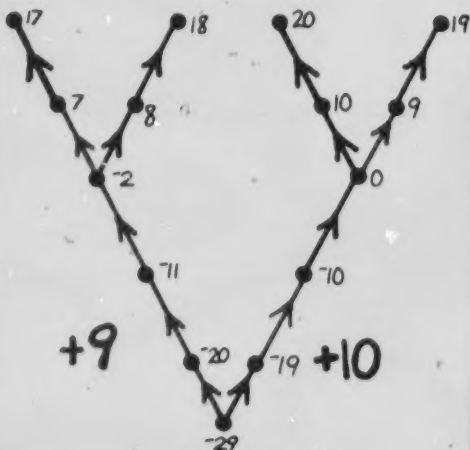
$$4 \times \boxed{1.25}$$

For each of the following pictures fill in the box in the upper right-hand corner with a label for the arrows. Then label the dots.



Tim is the largest number in this picture. Where is Tim?
Label his dot. Am I in this picture? no

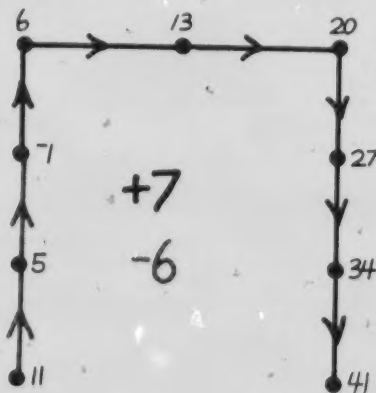
Label all the dots.

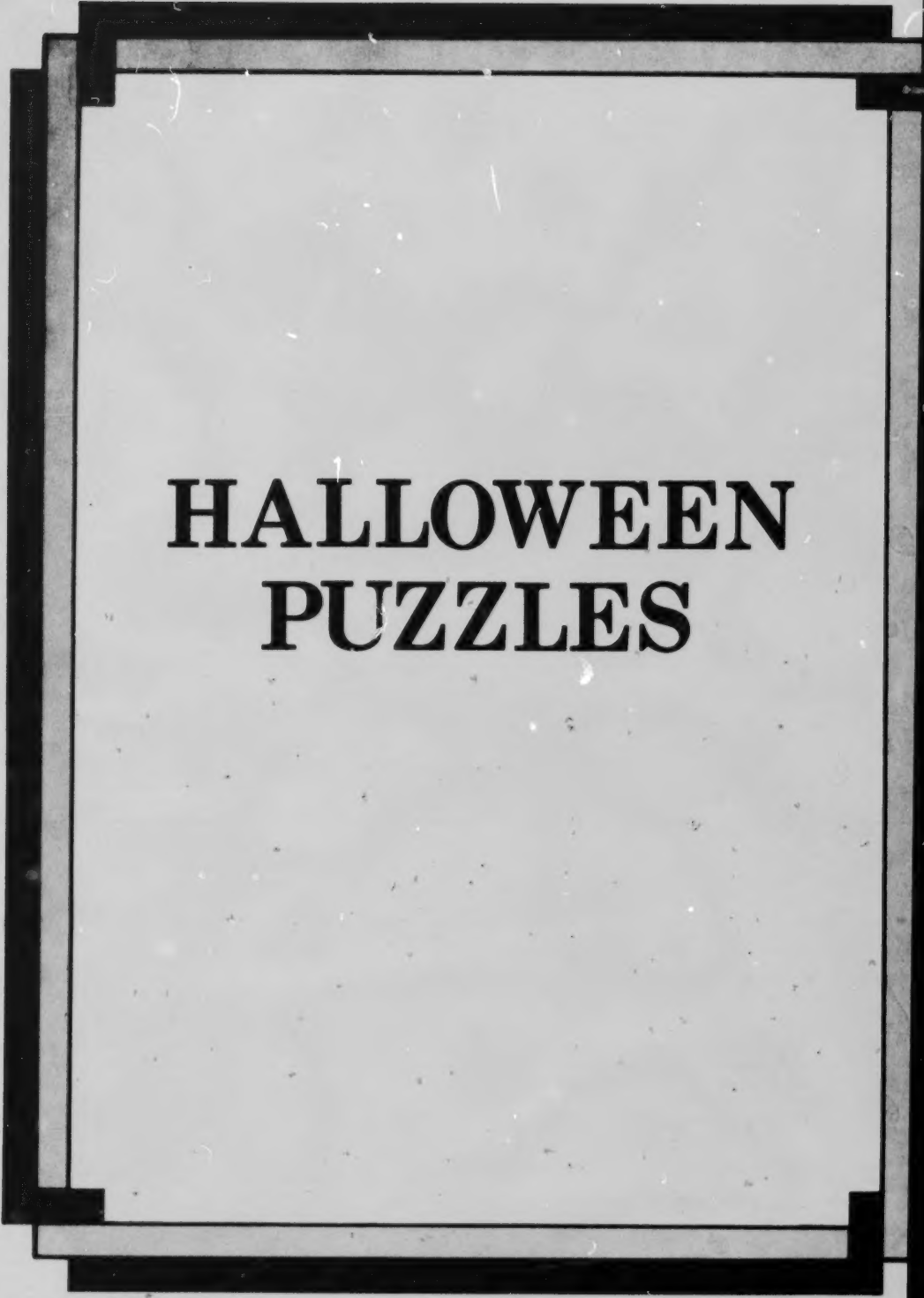


I am in this picture. So are Tim and some other numbers.

Did you find out where Tim and I are? Label our dots.

Label all the other dots.





HALLOWEEN PUZZLES

HALLOWEEN PUZZLES

Lesson 1

Distribute individual copies of the "Halloween Puzzles" story-workbook to the students and let them read any pages they wish for a few minutes.

Copy the picture from page 2 on the board and then read pages 1 and 3 to the whole group. Interrupt your reading on page 3 after line 7 and ask a student to point to the dots for 2, 8, 14, 20 and 11 on the board. Continue reading:

T: "If it were 10, then they would be __, __, __, 19."

Ask a student to point to the dot for 10 on the board.

T: What are the other numbers? Write their names in your book.
(Answer: 16; 22; 28; 19)

Follow similar steps for the next three sentences on page 3. Ask the students to read pages 5 and 7, and copy the picture from page 6 on the board.

T: If 13 jumps in the box, what are the other numbers?

Have a student point to the dot for 13 on the board and calculate the other four numbers. (19; 25; 31; 22)

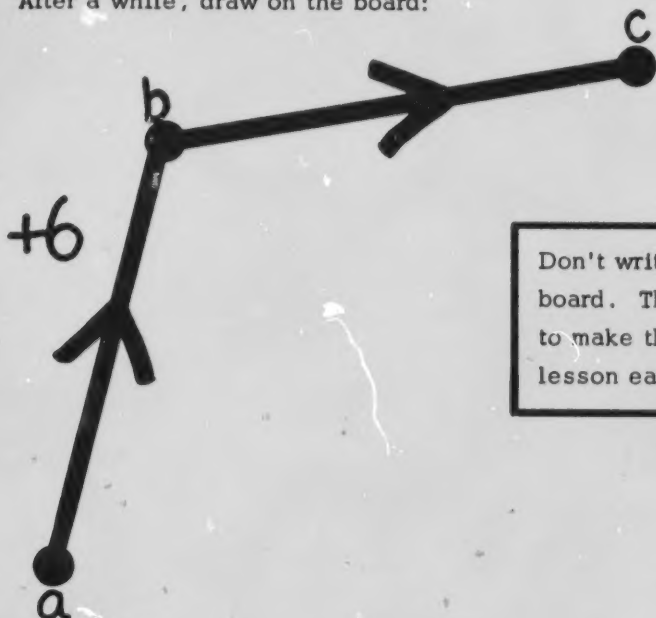
T: If 97 jumps in the box, what are the other numbers?

Have a student point to the dot for 97 on the board and calculate the other numbers. (103; 109; 115; 106)

T: What other numbers could jump in the box?

Follow the suggestions of the students and listen to their comments about the conclusion at the bottom of page 7. Then let them draw their own Halloween puzzle on page 8 and read the pages which follow.

After a while, draw on the board:



Don't write the letters on the board. They are written here to make the description of this lesson easier to follow.

T: What is the smallest number in this picture?
What is the largest number in this picture?

Have a student point to the smallest number (a) and to the largest number (c).

T: How much bigger is this number (point to c) than this one (point to a)?

S: 12

● Draw this green arrow:

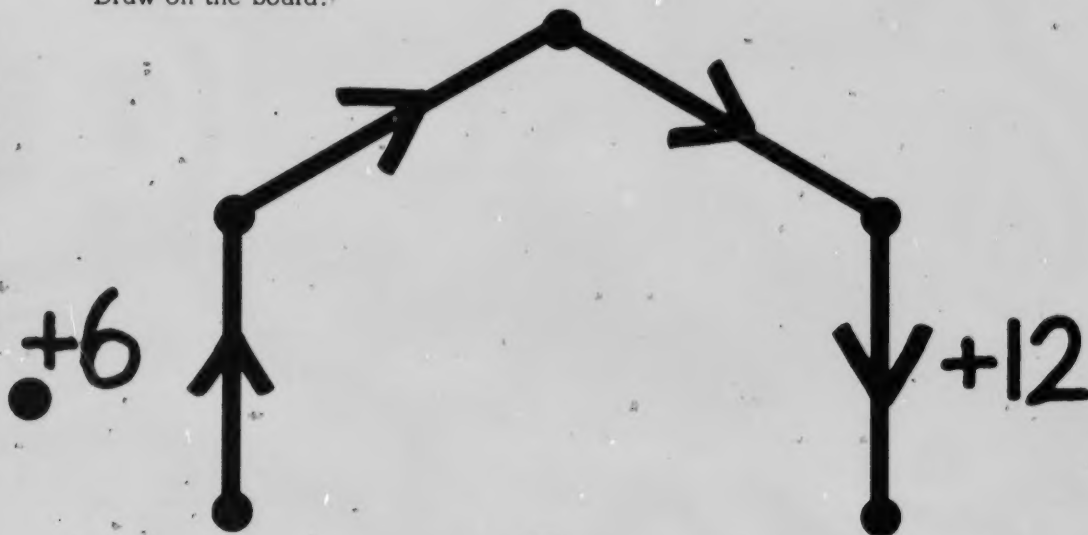


T: What could the green arrow be for?

S: $+12$

● Write " $+12$ " in green on the board.

Draw on the board:



T: Who could trace green arrows between some dots on this picture?

Have three students trace the three possible arrows.

Copy the picture from page 13 on the board and ask a student to trace a -20 arrow.

Ask the students to look at the pictures on pages 14-15 and try to explain why "this witch was teasing us." Encourage them to express their ideas, even if they do so awkwardly.

Read pages 16-18 to the whole group and let the students react.

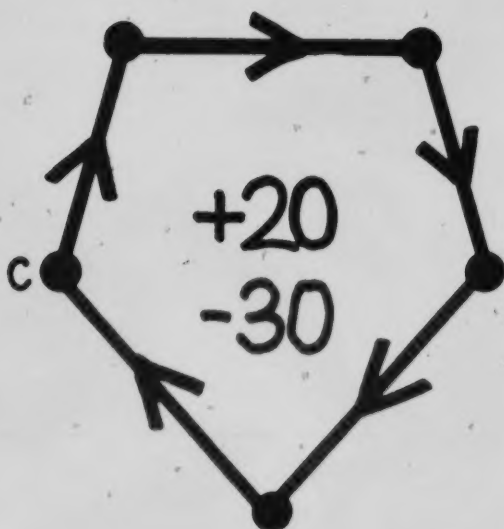
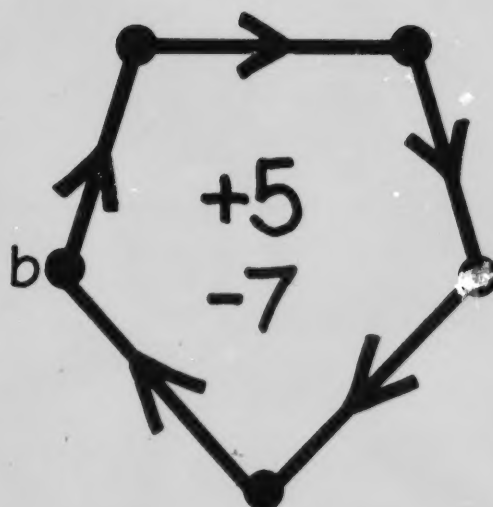
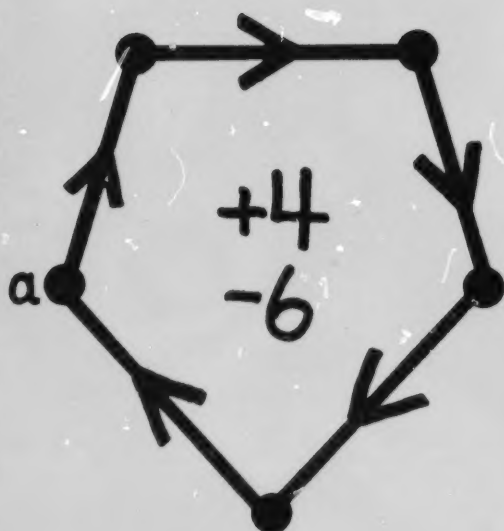
Ask them to draw their own Halloween picture on page 19.

Lesson 2

NOTE: The instructions for teaching Lesson 2 of "Halloween Puzzles" have been written with the assumption that there are calculators for the teacher and each member of the class. Since this may not be the case in every classroom, the following paragraph suggests a way of proceeding with only two calculators. This alternative procedure will allow you to include the part of the lesson which uses calculators, although it will be less effective, of course, than if each child has full use of a calculator. (Some students may have their own calculators; they should be encouraged to bring them to school for these lessons.)

The teacher must have a calculator and at least one student must have one. This student should be at the front of the class and, when the display is exposed, answer any of the questions relating to the calculator. Any of the children can answer the questions when the display is hidden. All of them should be encouraged to think just as carefully and stay just as involved as if they had a calculator with a hidden display. The student with the calculator can check all the answers. For each part of the lesson using the calculator, choose a new student, in order to give as many of them the experience as is possible.

Draw the pictures from the following page on the board and ask the students to copy them on a sheet of paper.



Don't write the letters on the board. They are written here to make the description of this lesson easier to follow.

T: At least one of these pictures was drawn by a witch. Try to find it (or them). Write "Witch" near the picture(s) drawn by a witch. Write "Nonwitch" near the picture(s) not drawn by a witch.

When many students have identified the "witch picture," ask all the students to take out their hand-calculators.

T: Let's choose a number for this dot. (Point to a.)

S: 55

T: Enter "55" in the calculator and hide the display (with your hand or a small piece of paper). Now let's follow the first arrow picture.

Press:

+	4	+	4	+	4	-	6	-	6	=
---	---	---	---	---	---	---	---	---	---	---

 .

Give the instructions slowly by pointing successively to each red arrow and to each blue arrow.

T: What number should be on your display now? Don't look.

Most of the students will probably answer "55".

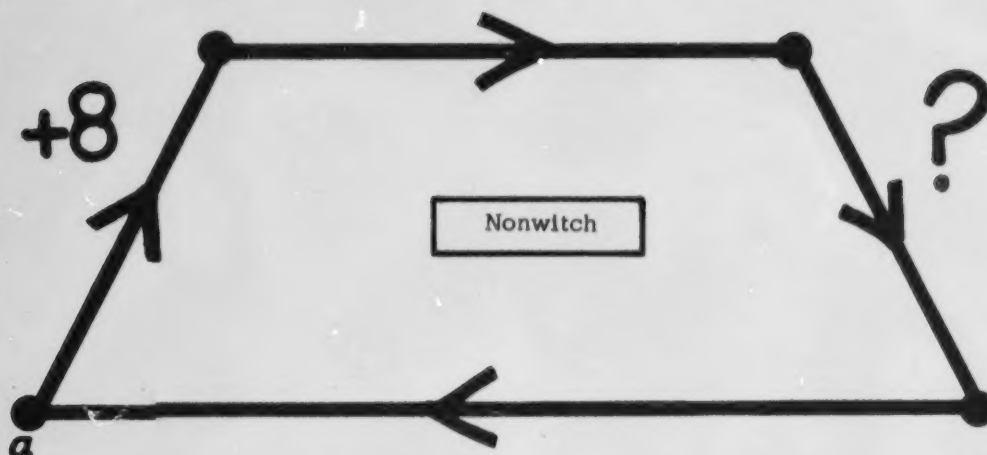
T: How do you know?

Let one student explain it and tell all to be sure that 55 is really on the display.

Follow a similar procedure with the "witch picture," starting from b with a number chosen by a student. They will probably predict that the ending number (b) has to be 1 more than the starting number (b) and therefore it must be a witch picture.

Follow a similar procedure with the third picture (which is a "nonwitch picture.")

Draw on the board:



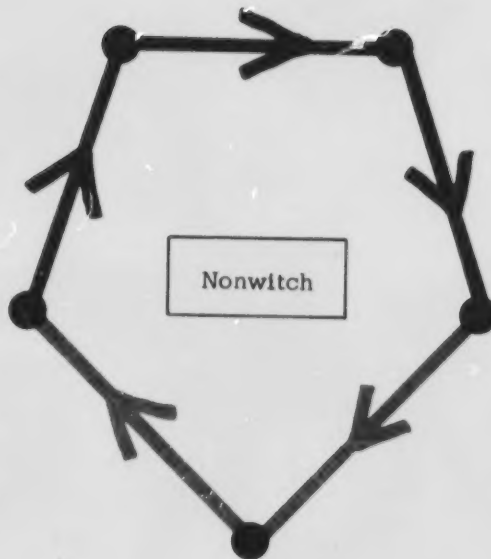
Don't write the letter on the board. It is written here to make the description of this lesson easier to follow.

T: This picture was not drawn by a witch. It has to be correct for any number we put here (point to the dot at a). Label the blue arrow.

Have a student explain why the blue arrow could be for -24 . Write " -24 " in blue on the board.

Do a similar exercise with the next three pictures:

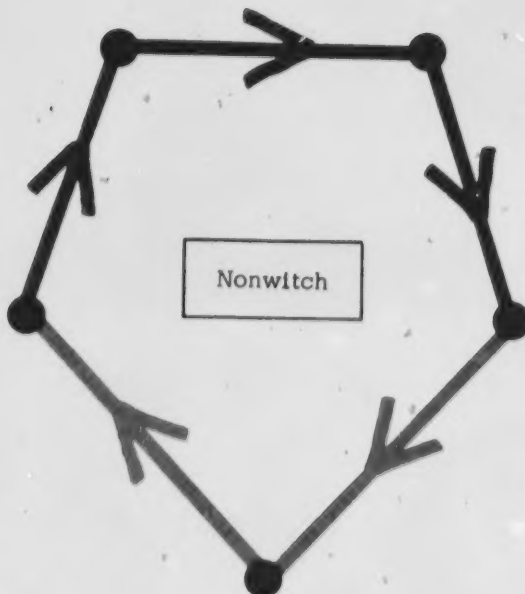
+100



?

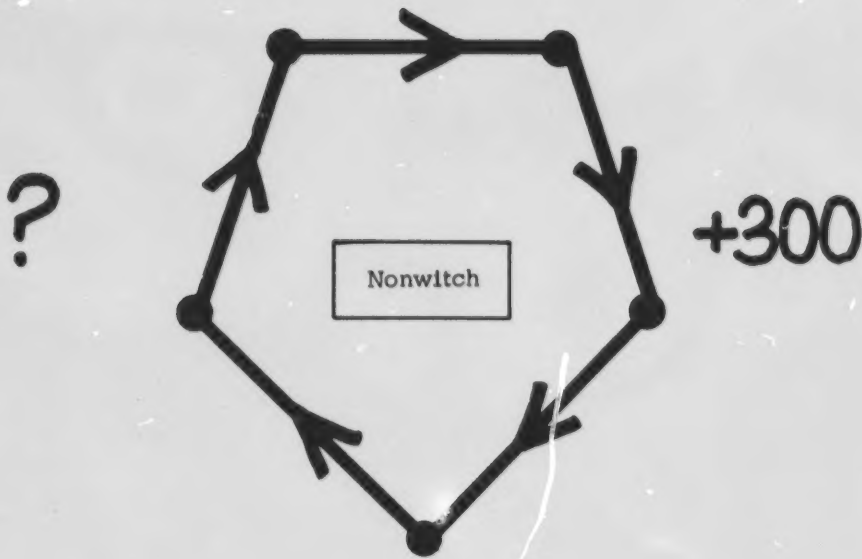
(Answer: -150)

?



+6

(Answer: -4)



(Answer: **-200**)

Towards the end of the lesson; ask the students to draw their own "witch picture" and their own "nonwitch" picture.

Lesson 3

Don't give the students their copy of the "Halloween Puzzles" story-workbook for the moment. Copy the picture from page 22 on the board.

Read page 23 to the whole group and ask them to explain why the secret number cannot be 2, 5, 8, 10, 15 or 50.

Ask them to write their guess on a sheet of paper and to keep it secret.

Complete the picture on the board as it is on page 24 and read page 25 to the whole group.

T: What is your guess for the secret number and its friend?

Each time a student makes a guess, ask the other students to check it.

Give each student his or her copy of the "Halloween Puzzles" story-workbook and ask them to look at pages 26 and 27.

T: Why can you draw a $4x$ arrow from the secret number to its friend x ?

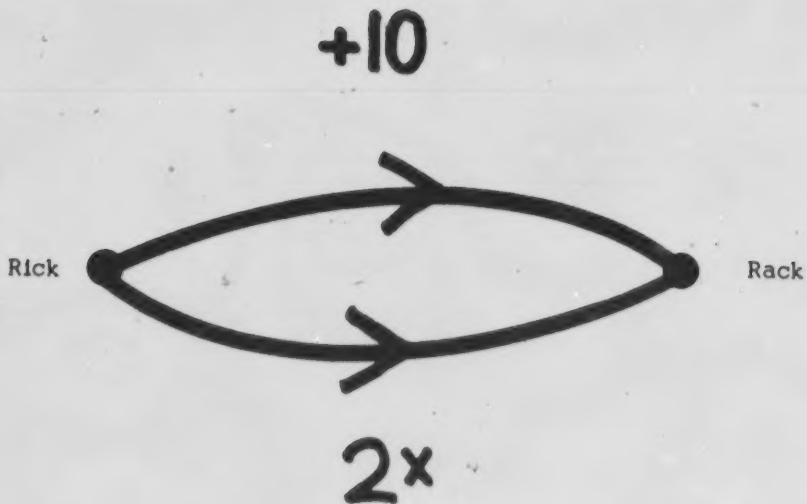
Have a student explain why.

T: Why can you draw a $+60$ arrow from the secret number to its friend x and a -60 arrow from x to the secret number?

Have a student explain why.

Let the students read and complete pages 29-31. They will conclude that the secret number is 20 and its friend x is 80.

Draw on the board:



T: Rick and Rack are two secret numbers. Try to find out who they are.
Could Rick be 2?

S: No, because $2 + 10 = 12$ and $2 \times 2 = 4$. Rack cannot be both 12 and 4.

T: Could Rick be 5?

S: No, because $5 + 10 = 15$ and $2 \times 5 = 10$. Rack cannot be both 15 and 10.

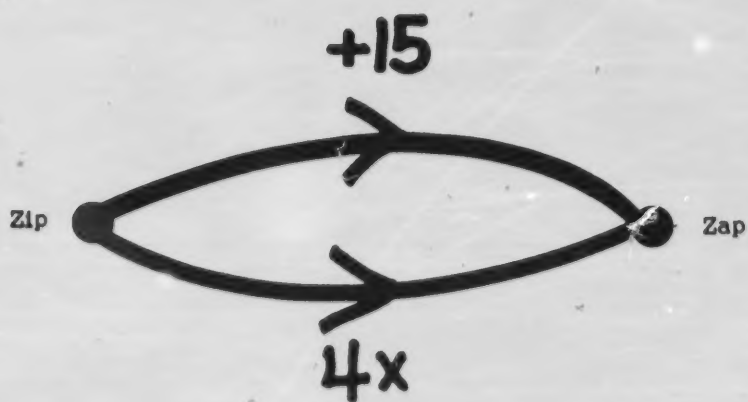
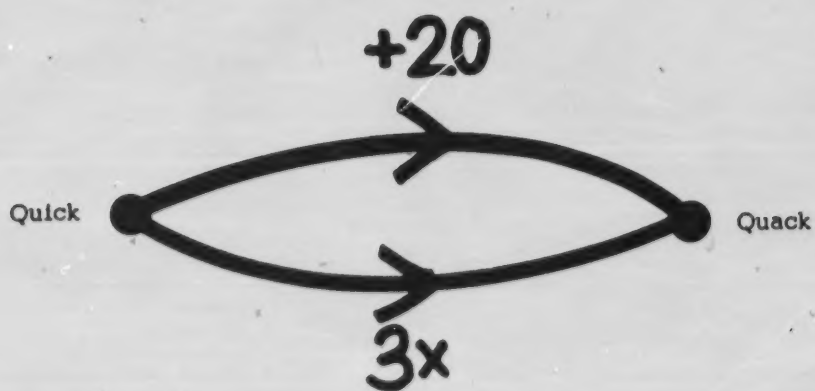
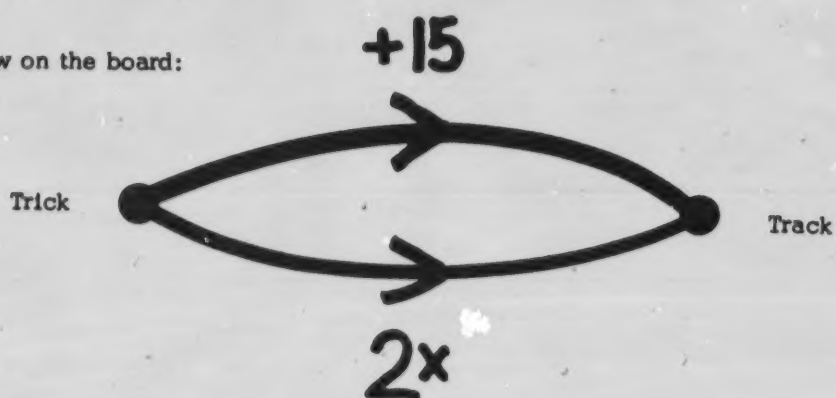
T: Could Rick be 20?

S: No, because $20 + 10 = 30$ and $2 \times 20 = 40$. Rack cannot be both 30 and 40.

T: Who is Rick? Who is Rack?

S: Rick is 10 and Rack is 20 because $10 + 10 = 20$ and $2 \times 10 = 20$.

● Draw on the board:



Let the students try to solve each of these puzzles by themselves.

After a while, have 3 students explain their answers:

Trick is 15 and Track is 30 because $15 + 15 = 30 = 2 \times 15$

Quick is 10 and Quack is 30 because $10 + 20 = 30 = 3 \times 10$

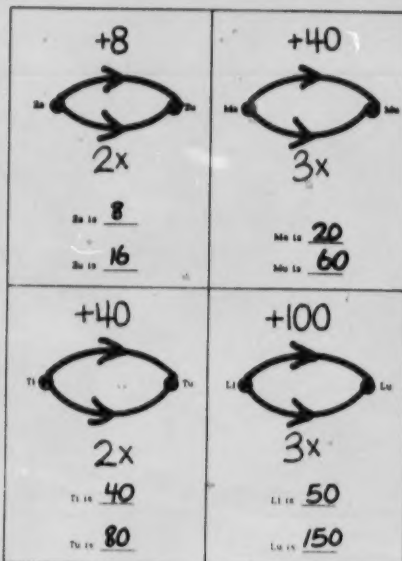
Zip is 5 and Zap is 20 because $5 + 15 = 20 = 4 \times 5$

Worksheets N48 and N49 are available for the rest of the period.

Name _____

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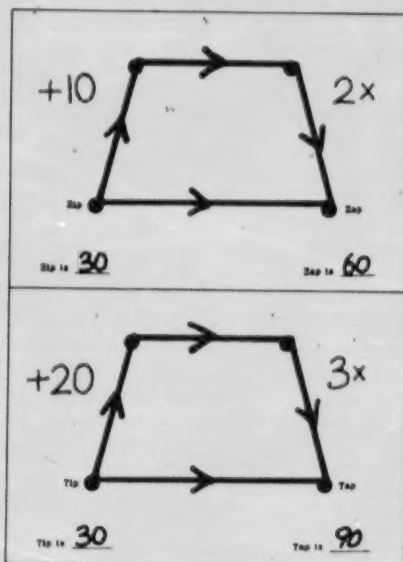
N48



Name _____

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N49



Halloween Puzzles

3

It's a number! But which number?

"If we knew that, some of us would take our places in this picture," answered the numbers.

For instance,

"If it were 2, then the other four dots would be 8, 14, 20, and 11 because $2 + 6 = 8$; $8 + 6 = 14$; $14 + 6 = 20$; $20 - 9 = 11$; and $11 - 9 = 2$."

"If it were 10, then they would be 16, 22, 28, 34."

"If it were 15, then they would be 21, 27, 33, 24."

"If it were 37, then they would be 43, 49, 55, 46."

"If it were 125, then they would be 131, 137, 143, 134."

You can play this game many more times if you would like to.

7

"If I jump into the box," says 0, "then the other dots are 6, 12, 18, and 9."

"If I jump into the box," says 13, "then the other dots are 19, 25, 31, 22."

"If . . . If . . . If . . . We could go on playing this game forever," observe the numbers. "Each of us could jump into the box."

They are silent for a while.

"Without any doubt, that is the reason why the box is empty," they conclude.

11

"There are six numbers in the picture," murmurs 0.

"I wonder if I can guess who they are."

He looks at the picture and seems to be worried.

What about you?

"If I were this dot , " says 0, "then the other dots

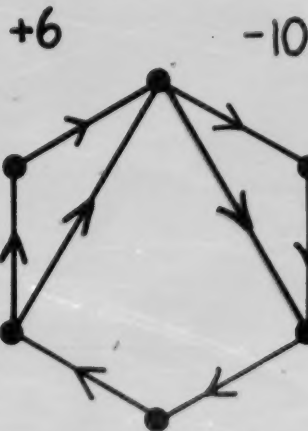
would be 6, 12, 18, 24, 14. That's strange."

Why is it strange? Write what you think below.

If this dot is for 0, then by following the arrows we see that it also has to be for 4. But one dot cannot represent two different numbers at the same time.

12

0 adds two new arrows to the picture on the ground.

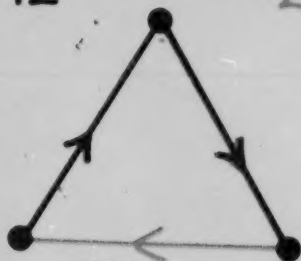


What could these green arrows be for?

The green arrows could be for + 12.

+12

-20



"Look," says 0. "I have the feeling that this witch was teasing us."

Use this space to explain why.

Choose any number you wish for the dot at the lower left and follow the arrows. First you add 12 to your number; then add 12 to the result; then subtract 20 from the result. According to the picture you should get back to the number you started from, but you don't. Since one dot cannot represent two numbers at the same time, the picture doesn't work.

13

I am a number.

I am playing with three of my friends.

The first one is my double.

The second one is the double of my double.

What do you know about the third one?

(The third one is thirty less than the double of my double.)

Could I be 5? Why not?

No, because $4 \times 5 \neq 5 + 60$.

Could I be 10? Why not?

No, because $4 \times 10 \neq 10 + 60$.

Could I be 15? Why not?

No, because $4 \times 15 \neq 15 + 60$.

Could I be 50? Why not?

No, because $4 \times 50 \neq 50 + 60$.

Who am I? Write your guess below.

(Since only a guess is asked for here, no wrong answers are possible. Since the narrator of this story is the number 20 (see p. 27), the best guess is 20.)

23

My friend x whispers to me:

"I am 4 times you."

I am larger than you.

I am exactly 60 more than you."

Do you agree with x ?

Can you guess who x is?

Can you guess who I am?

Write your guess for me and for x below.

(Since the narrator is 20 and the narrator's friend x is 80 (see p. 27), these would be the best guesses. However, there are no wrong answers for this exercise.)

Check your guess by calculating the other two numbers.

25

Here is a clue to help you find who we are.

$$4 \times 20 = 20 + 20 + 20 + 20 = 80$$

$$20 + 60 = 20 + 20 + 20 + 20 = 80$$

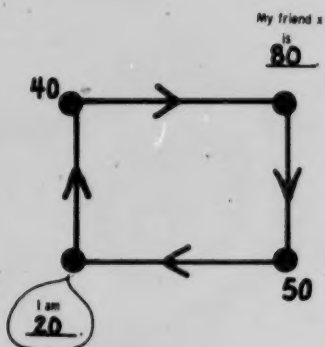
Who am I? Who is my friend x ? Write your answers below.

20

80

27

28

 $2x$ -30 

Who are the other two numbers who are playing with us?

Write their names on the picture.

29

Look at the picture on page 28 again.

Did you find the clue to this puzzle there?

Explain below.

The letters in the picture spell "twenty".

SUMMER CAMP

SUMMER CAMP

Lesson 1

Give a copy of the "Summer Camp" story-workbook to each student and ask them to read pages 1 - 5 and to study page 5 carefully.

Write on the board:

___; ___; ___; ___; 0; 1; 1; 2; 3; 5; 8; 13; 21; 34; 55; 89; ___; ___; ___

Point to "8" and ask:

T: What do you notice about 8?

S: $8 = 3 + 5$

Ask similar questions for: 21 ($8 + 13 = 21$); 2 ($1 + 1 = 2$); 34 ($21 + 13 = 34$); 89 ($34 + 55 = 89$).

T: If the numbers keep on going according to this pattern (rule), then in this sequence of numbers what is the next number to the right of 89? (Answer: 144, since $55 + 89 = 144$)

Ask the students to try to find at least the next six numbers and to write them on a sheet of paper.

Answers:

89	144	233	377	610	987
<u>+144</u>	<u>+233</u>	<u>+377</u>	<u>+610</u>	<u>+987</u>	<u>+1,597</u>
233	377	610	987	1,597	2,584

T: In the sequence of numbers that is written on the board, if the numbers to the left of 0 are also determined by this pattern, then what is the first number to the left of 0?

If no student finds it, write on the board:

$$\square + 0 = 1$$

S: It's 1, because $1 + 0 = 1$.

T: What number goes in this blank?

Point to the first blank to the left of "1".

___; ___; ___; 1; 0; 1; 1; 2; 3; 5; 8; ...

S: -1 because $-1 + 1 = 0$.

Have the students find at least six more numbers to the left of -1. Add as many blanks as needed to the left of the list on the board. (Answer: ...; -21; 13; -8; 5; -3; 2; -1; 1; 0; 1; 1; 2; 3; 5; 8; ...)

T: What do you notice about these numbers?

Some possible answers:

S: There is a pattern.

S: They are successively positive and negative.

S: They are the same numbers as in the Fibonacci Song, but alternately positive and negative.

Have the students work on the problem given on pages 6 - 7.

T: Do you see which boat 40 and 100 are in?

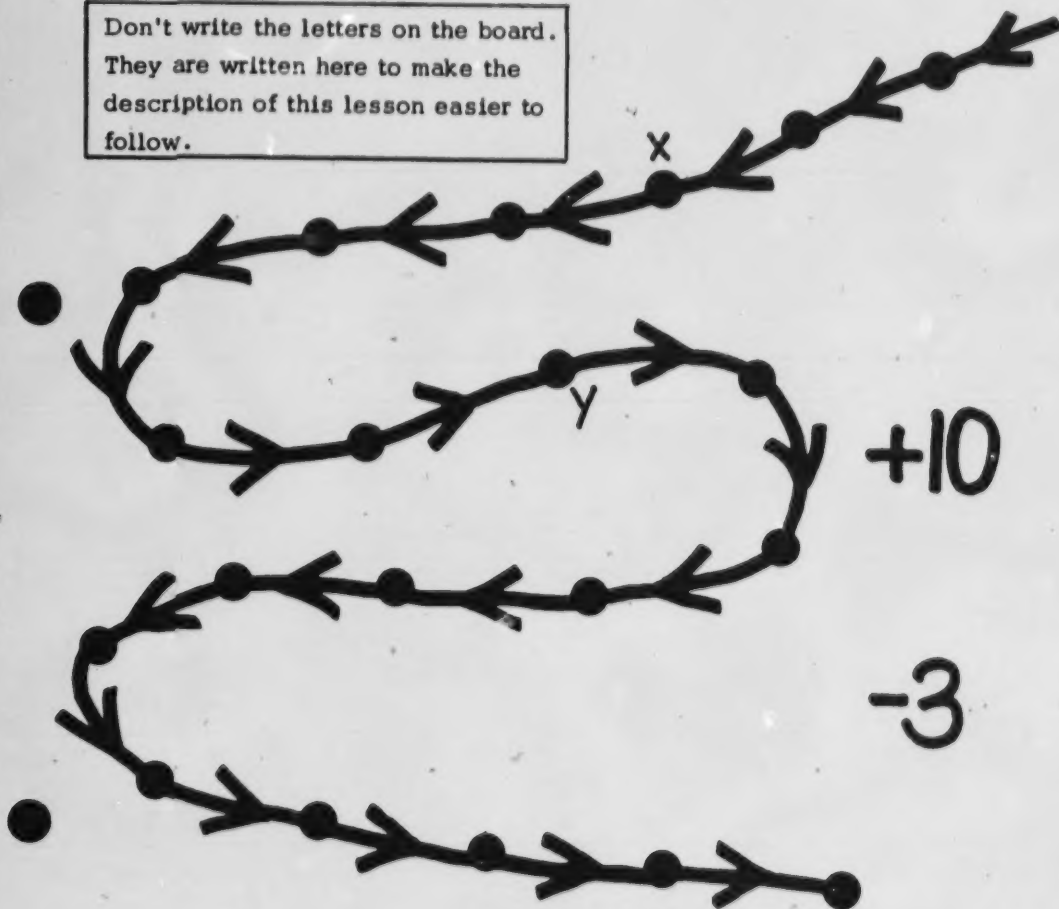
S: In the boat with the blue arrow.

T: How do you know?

S: Because 100 is 60 more than 40.

Have the students label all the dots of the picture and continue working on the other problems by themselves. After a while, draw the picture from pages 8 - 9 on the board:

Don't write the letters on the board.
They are written here to make the
description of this lesson easier to
follow.



T: Did you find 40 and 100 on pages 8 - 9?

Have a student explain that 40 is at x and 100 at y because this number (y) is 60 more than this number (x). Don't label the other dots on the board.

Have the students work on their own for a while. Then say:

T: Look at pages 10 - 11, even if you haven't finished the other problems.

What is the smallest number in this picture?

S: The starting number of the road.

T: What is the largest number in this picture?

S: The ending number of the road.

T: How many blue +5 arrows are there between these two numbers?

S: 10 blue arrows.

T: How many red +1 arrows?

S: 10 red arrows.

T: How much bigger is this number (point to the largest one) than this number (point to the smallest one)?

S: 60 bigger.

Have the students conclude that 40 is the starting number of the road and 100 is the ending number.

Have the students work at their own pace for the rest of the period.

Lesson 2

Give each student his or her copy of the "Summer Camp" story-workbook and ask them to open the book to pages 12 - 13.

T: Do you remember who is telling the story?

S: The number 100.

T: Who is 100's best friend?

S: The number 40.

T: How much bigger is 100 than 40?

S: 60 bigger.

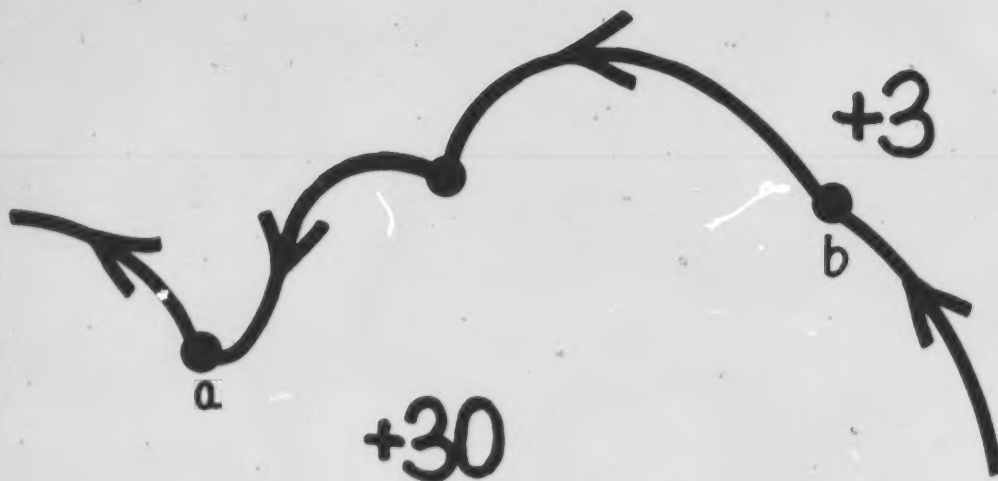
T: What game are 100 and 40 playing on pages 12 and 13?

S: Hide and Seek.

T: Can you discover where they are hiding?

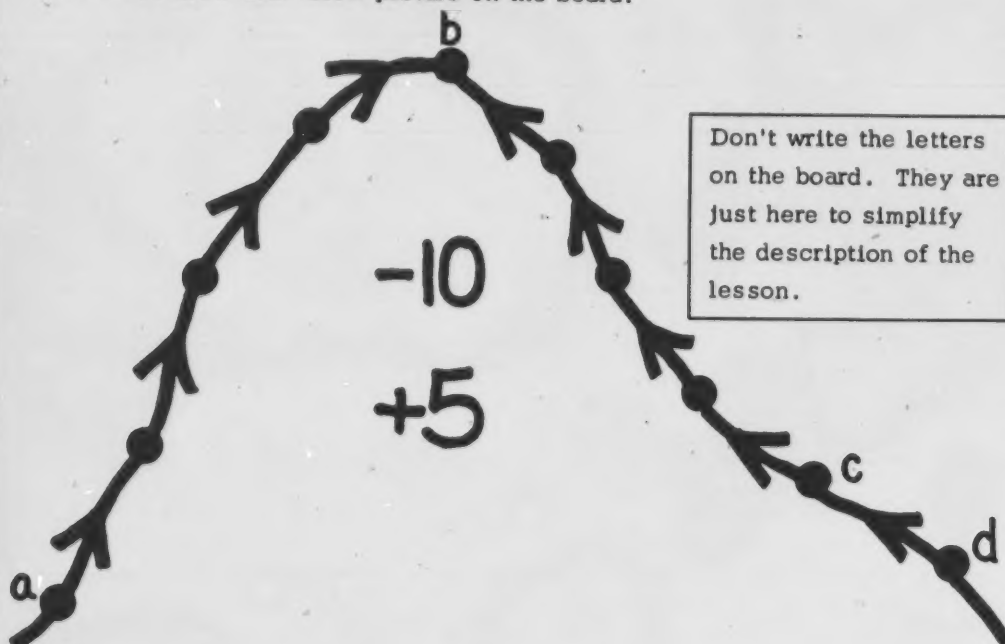
The blue arrows are for +30 and the red arrows are for +3.

Have a student explain that 100 is inside the box and 40 is inside one of the trash cans because the two blue arrows show that the number (a) inside the box is 60 more than the number (b) inside the trash can.



Have the students label all the dots in this picture and then continue working at their own pace on the other problems in the book.

If at some moment many students are stuck on the problem given on pages 14 - 15, draw this arrow picture on the board:



Point to a and b and ask:

T: Which is the smaller of these two numbers?

S: The one at the top. (b)

T: How much smaller is this number (point to b) than this one (point to a)?

S: 40 smaller.

Point to b and d and ask:

T: Which is the smaller of these two numbers?

S: The one at the bottom. (d)

T: How much smaller is this number (point to d) than this one (point to b)?

S: 25 smaller.

Don't concentrate any more on a, b or d, but consider the whole picture:

T: Which is the largest number in the picture?

S: The bottom one on the left. (a)

T: Which is the smallest number in the picture?

S: The bottom one on the right. (d)

T: How much smaller is this number (point to d) than this one (point to a)?

S: 65 smaller.

T: Can you find a number that is exactly 60 less than this one (point to a)? Come to the board and show us where it is.

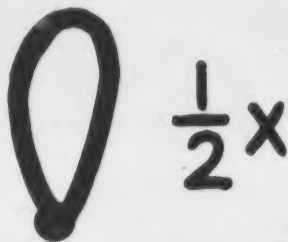
S: It's here just above the smallest number (pointing to c).

T: Do you see where 100 and 40 are in this picture? Come to the board and show us where they are. (Answer: a is 100 and c is 40.)

Have the students label (in their books) all the dots in this picture and continue working at their own pace. At some moment, many students will probably be stuck on the problem given on pages 18-19. Ask all the students to stop their individual work for a little while and to open their books to pages 18-19.

T: There is a red loop in this picture. What does it mean?

Draw the loop on the board:



S: Half of this number is the same number.

T: Could this number be 100?

S: No, because $\frac{1}{2} \times 100 = 50$.

T: Could this number be 40?

S: No, because $\frac{1}{2} \times 40 = 20$.

T: Could this number be 20?

S: No, because $\frac{1}{2} \times 20 = 10$.

T: What number could this dot be for?

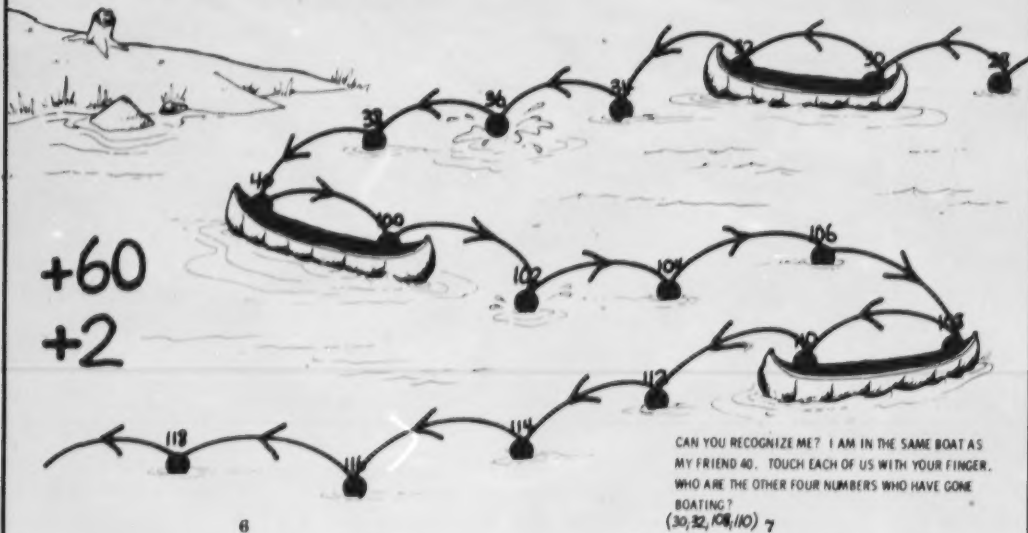
S: 0, because $\frac{1}{2} \times 0 = 0$ and 0 is the only number which has this property.

Have the students conclude that 0 is the guide of the numbers who are visiting the Caverns of Ma Kro.

Let them work again on their own problems and at their own pace for the rest of the period. But if at some moment a sizeable number of students are struggling with the same difficulty, don't hesitate to give them a short collective explanation.

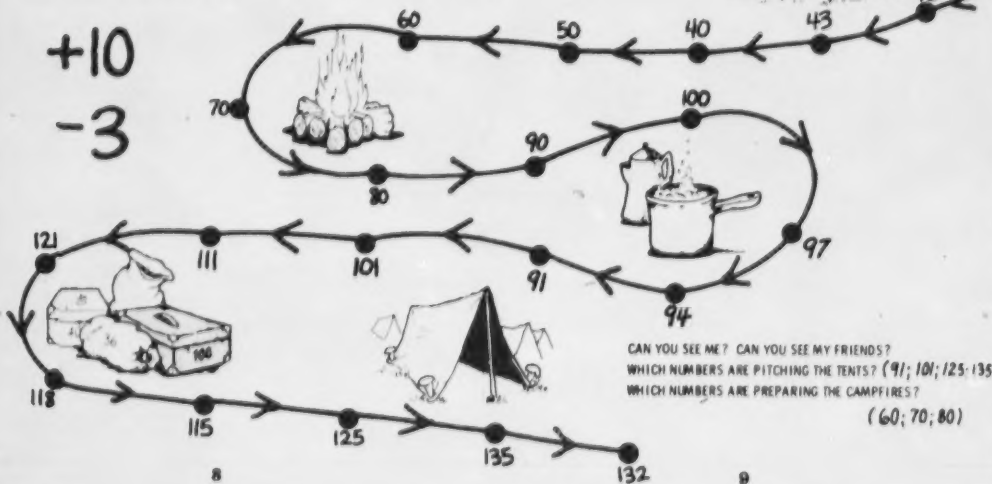
Summer Camp

The sun is blazing down and it is very hot. We are thirsty. After two hours of driving, we stop near a lake and we all take a swim or go boating.



CAN YOU RECOGNIZE ME? I AM IN THE SAME BOAT AS
MY FRIEND 40. TOUCH EACH OF US WITH YOUR FINGER.
WHO ARE THE OTHER FOUR NUMBERS WHO HAVE GONE
BOATING?
(30, 32, 108, 110) 7

We have arrived. We are all busy pitching camp. Some other numbers are helping 40 to fetch some logs. I am working on the cooking team.



CAN YOU SEE ME? CAN YOU SEE MY FRIENDS?
WHICH NUMBERS ARE PITCHING THE TENTS? (91; 101; 125; 135)
WHICH NUMBERS ARE PREPARING THE CAMPFIRE?
(60; 70; 80)

+5

+1

Tonight is our first night at camp. We are sleeping in an old barn with some other numbers. 40 is very sad because his sleeping bag is a long way from mine.

CAN YOU SEE WHERE MY SLEEPING BAG IS? WHERE IS MY FRIEND'S SLEEPING BAG? HOW MANY NUMBERS ARE SLEEPING BETWEEN THE TWO OF US? WHO ARE THESE NUMBERS?

(There are 19 numbers in this picture between 40 and 100.)

10

11

This morning, we are playing hide-and-go-seek in the yard outside the barn where we sleep. O has to find us. I don't think he'll ever manage to find me or my friend 40.

CAN YOU DISCOVER WHERE WE ARE HIDING? CAN YOU RECOGNIZE ALL THE NUMBERS WHO ARE TAKING PART IN THE GAME?

+3
+30

12

13

We are climbing over a high mountain.

OUR GUIDE HAS ALREADY REACHED THE SUMMIT.
DO YOU KNOW WHICH NUMBER HE IS? (60)

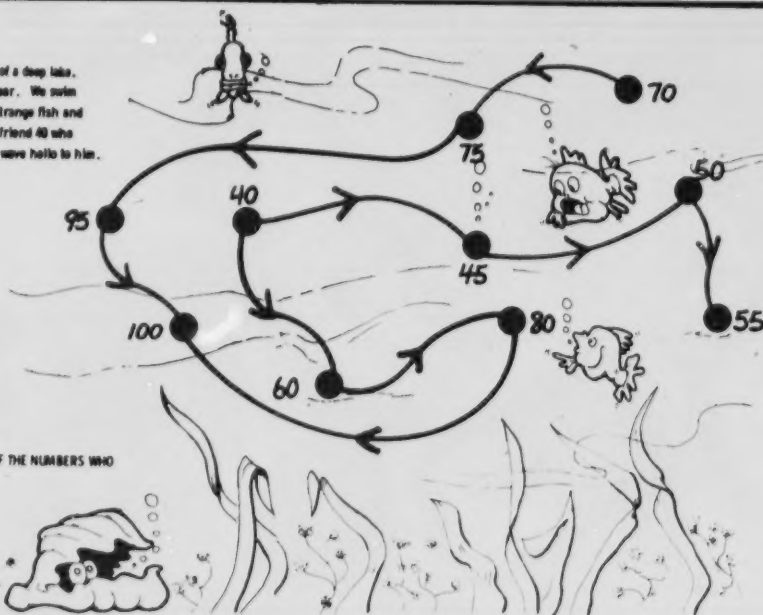
MY FRIEND AND I ARE NOT ROPED TOGETHER.
TRY TO FIND OUT WHERE WE ARE.

-10
+5



We are exploring the bottom of a deep lake.
We are wearing our diving gear. We swim
along in a world filled with strange fish and
plants. I catch sight of my friend 40 who
is an excellent swimmer. I wave hello to him.

+5
+20

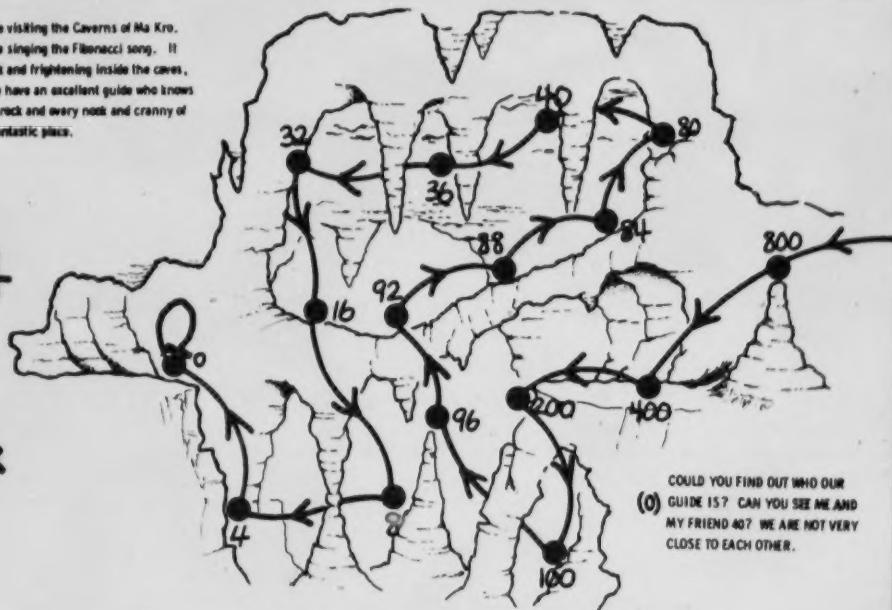


CAN YOU RECOGNIZE EACH OF THE NUMBERS WHO
ARE WITH US?

We are visiting the Caverns of Ma Kro.
We are singing the Fibonacci song. It
is dark and frightening inside the caves,
but we have an excellent guide who knows
every rock and every nook and cranny of
this fantastic place.

-4

$\frac{1}{2}x$



COULD YOU FIND OUT WHO OUR
GUIDE IS? CAN YOU SEE ME AND
MY FRIEND 40? WE ARE NOT VERY
CLOSE TO EACH OTHER.

18

19

One of our leaders, the number 1,000,000,000,
has organized a big pole-climbing contest. My
little friend 40 is very good at climbing poles. He
doesn't seem the least bit afraid as he jumps from
the top of one pole to another. I am there too,
but I'm not able to climb very high.

+5

+12⁻⁴

-9



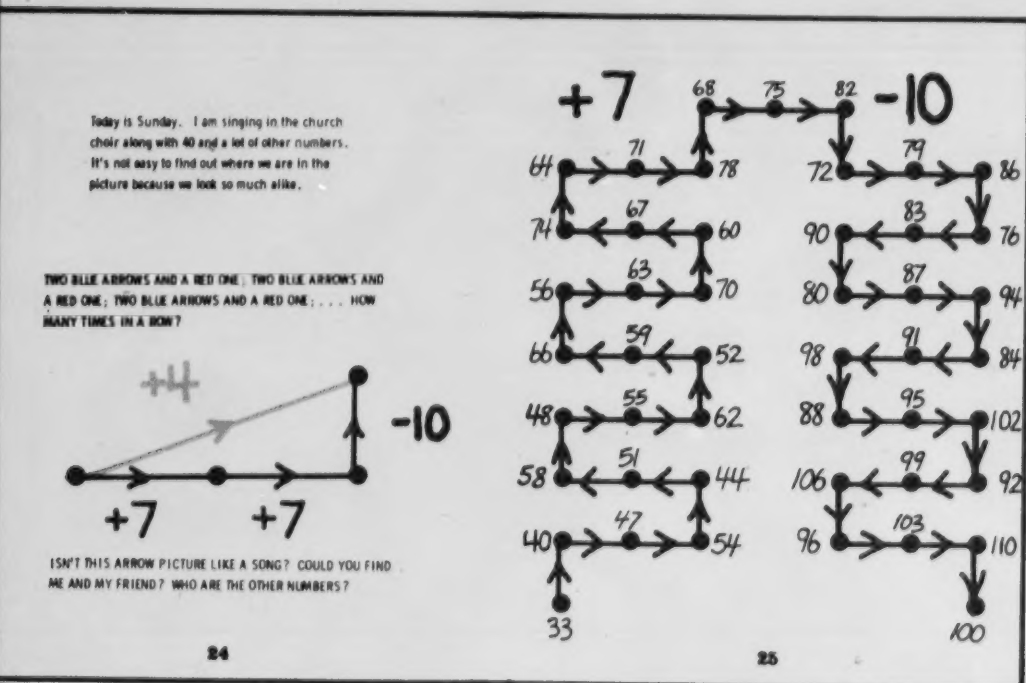
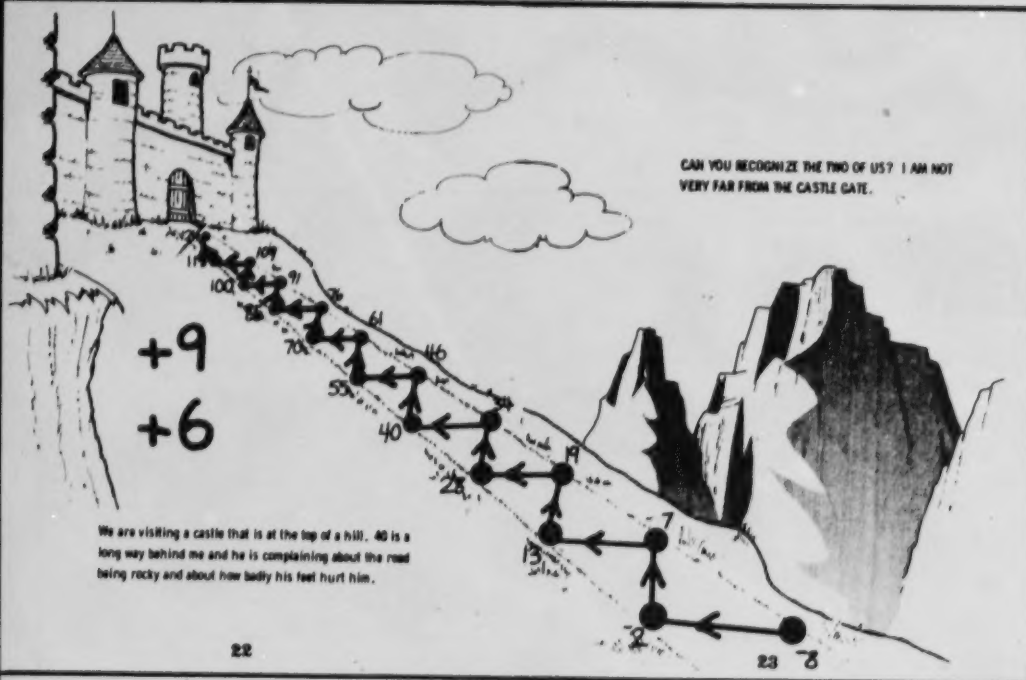
WHERE AM I? WHERE IS MY FRIEND?

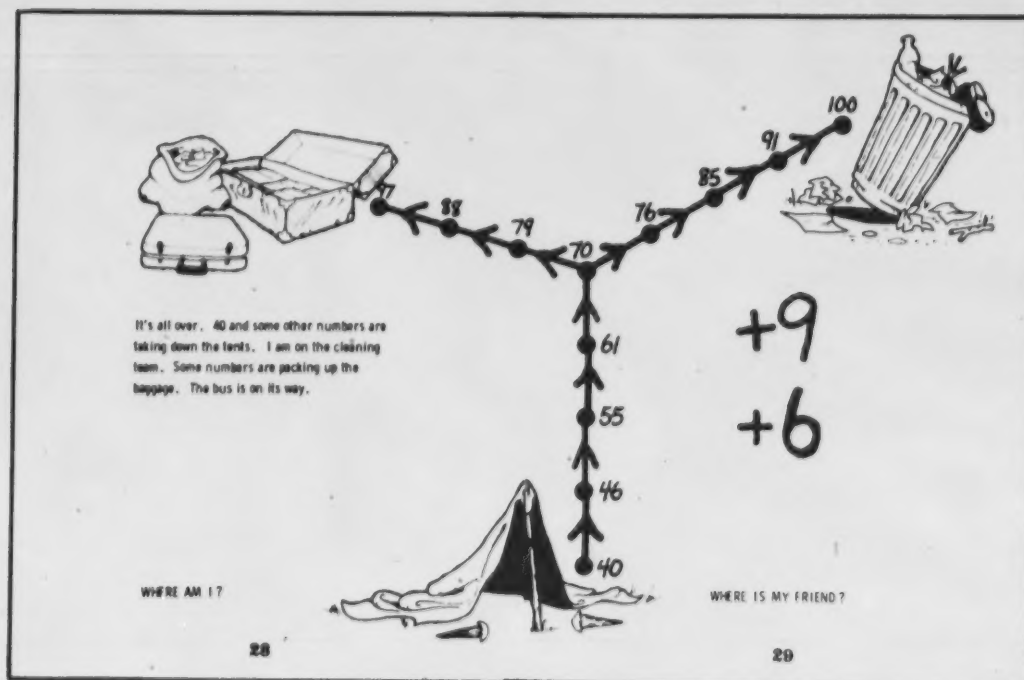
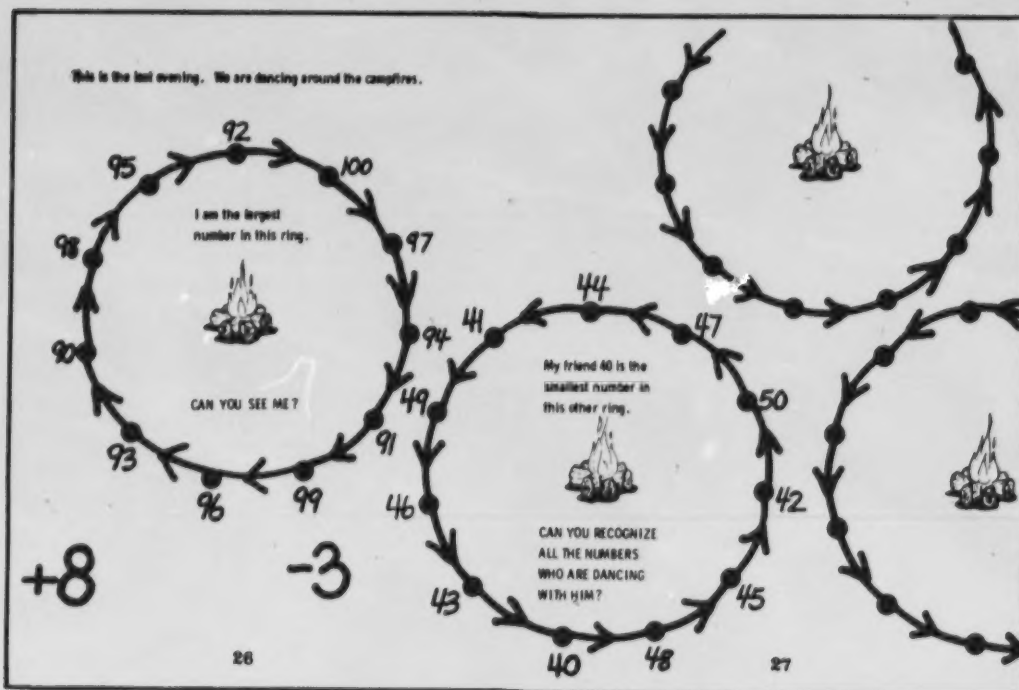
20

21

65

145





**A COLLECTION
OF
PROBLEMS #2**

COLLECTION OF PROBLEMS #2

This workbook series has three levels of difficulty (*, ** and ***). It is suggested that all the students begin with level * and continue through as many of the workbooks as they can handle during the three lessons scheduled for this series.

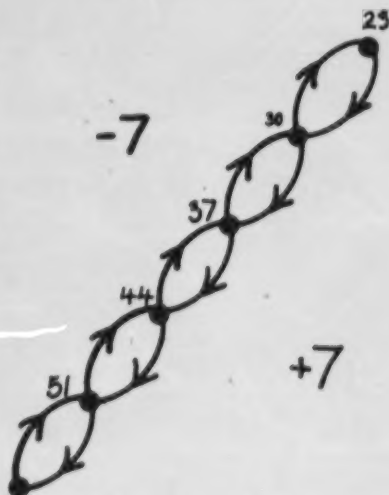
The series has been designed to encourage the students to work independently; therefore no collective introductory lesson is necessary. Though there are many different types of problems, all of them will be familiar to the students because of previous work they have done in one or another of the strands.

If, at any time, you observe that many students are having difficulty with a particular page, by all means, stop the independent work while you clarify the problem collectively. Then let them resume their individual work as you continue to observe and give individual assistance.

A Collection of Problems #2 *

2

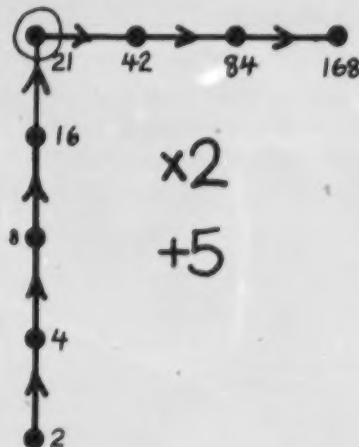
Label the dots.
What could the blue arrows be for? Use blue to write your answer.



3

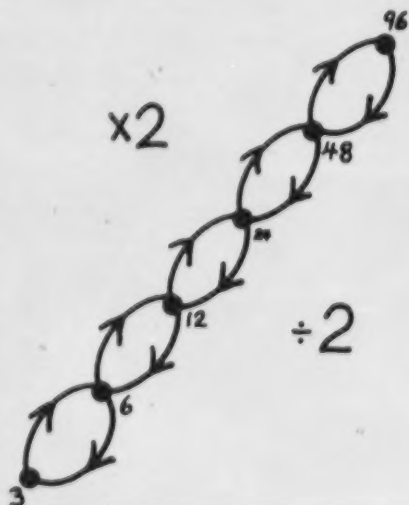
Dack is the only odd number in this arrow picture.
Label all the dots.

Who is Dack? 21. Circle his dot.



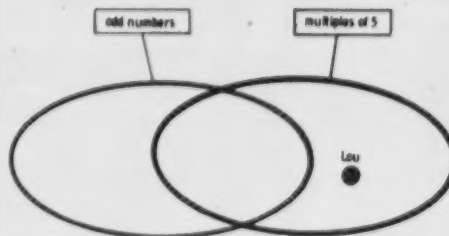
4

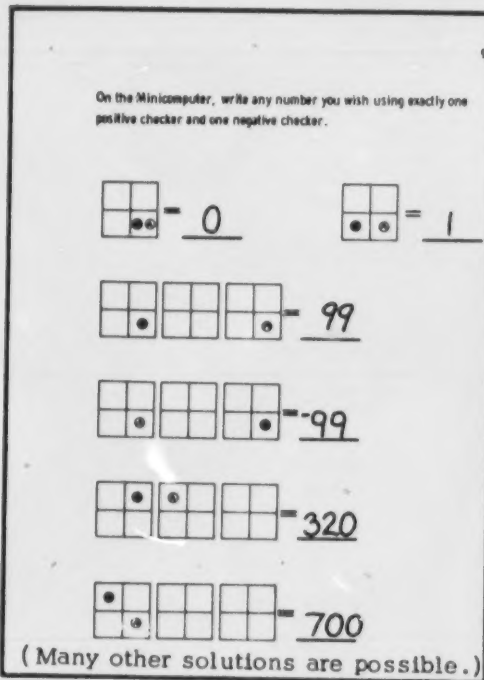
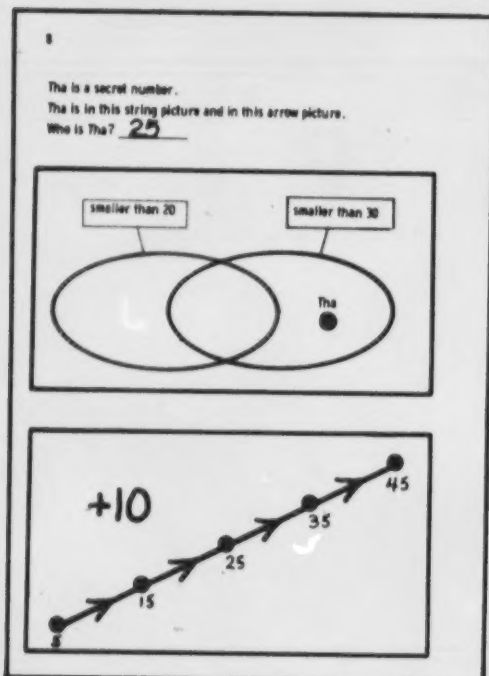
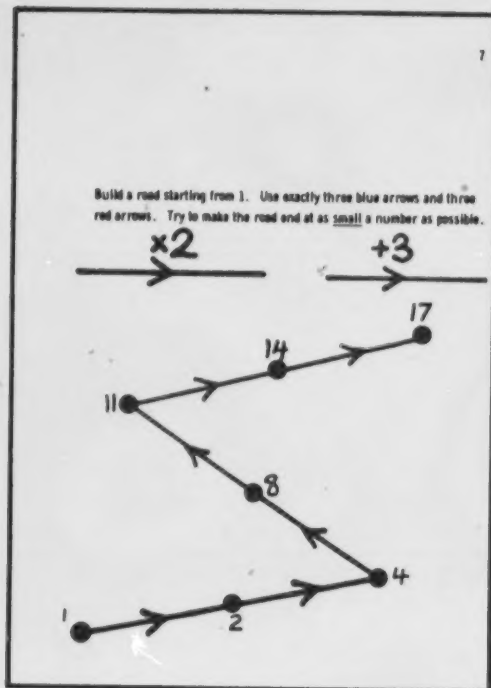
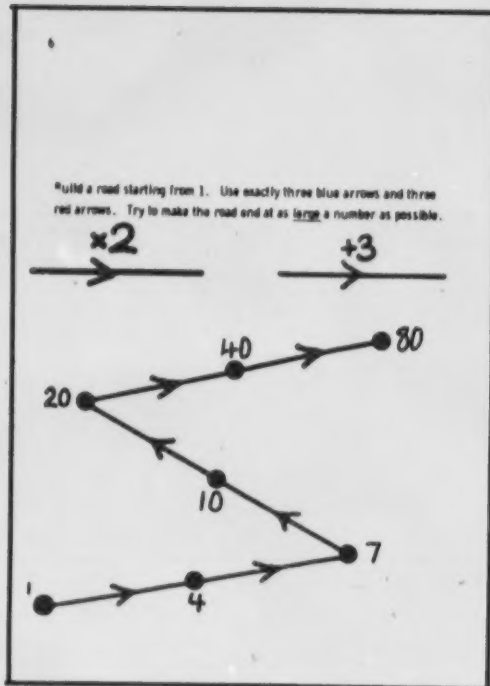
Label the dots.
What could the red arrows be for? Use red to write your answer.



5

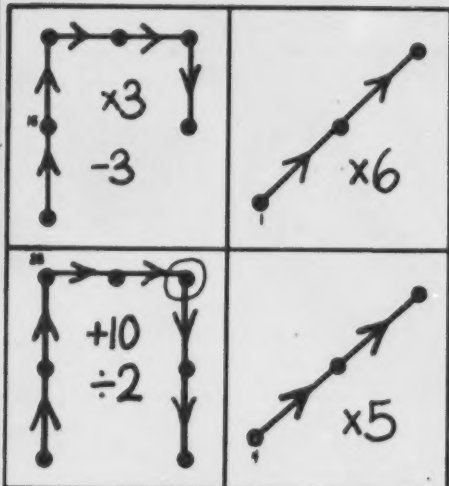
Lou is a secret number.
Lou can be shown on the Minicomputer with exactly two positive
checkers on the ones' board.
Lou is in this string picture.
Who is Lou? 10





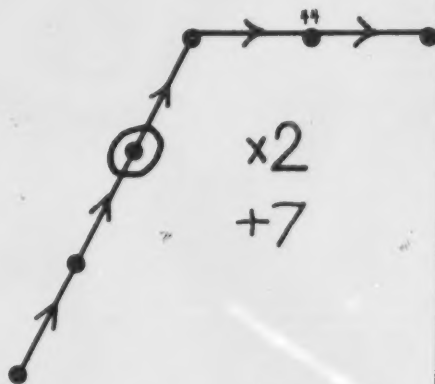
10

7 is in exactly one of these pictures.
Try to find where 7 is. Circle its dot.



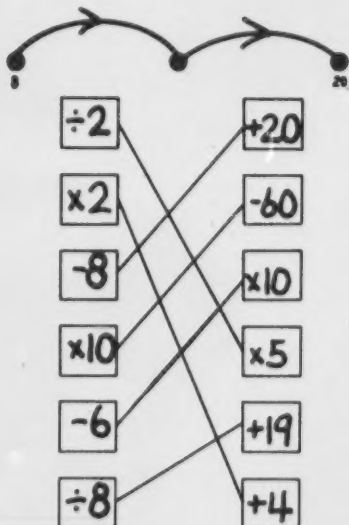
11

Rha is the only multiple of 3 in this arrow picture.
Who is Rha? 15. Circle its dot.



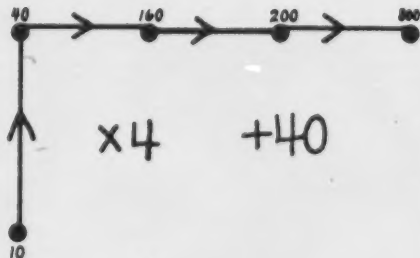
12

Pair the tags:



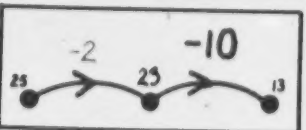
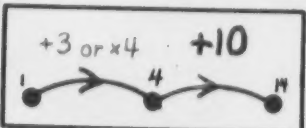
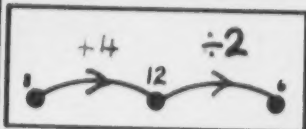
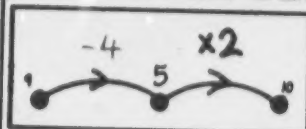
13

Ker is a secret number.
Ker cannot be put on the Minicomputer with exactly one checker.
Ker is in this arrow picture.
Who is Ker? 160



14

In each picture, label the dots. Then label the blue arrows.

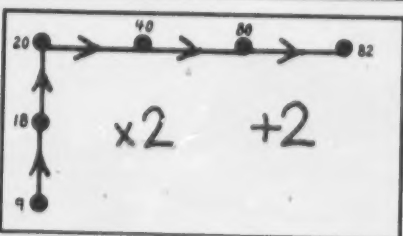
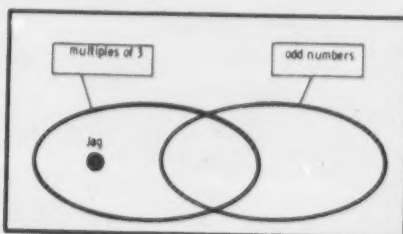


15

Jag is a secret number.

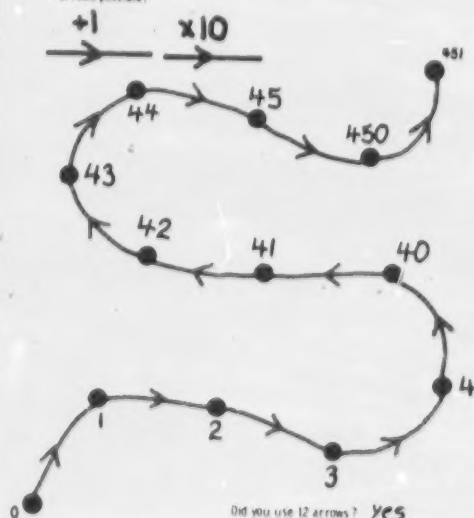
Jag is in this string picture and in this arrow picture.

Who is Jag? 18



16

Build a blue-rad road from 0 to 451 using the smallest number of arrows possible.



Did you use 12 arrows? Yes

73

151

A Collection of Problems #2 **

2

Pair the tags:

$+6$ $+20$
 $+4$ $5x$
 -4 $+28$
 $2x$ $+24$
 $+8$ $+10$
 -8 $+22$
 $3x$ $+12$

3

$\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} = 6$
 $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} = 70$
 $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} = 6$
 $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} = 70$
 $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} = 6$
 $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} = 30$
 $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} = 6$
 $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} = 30$
 $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} = 19$
 $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} = 99$
 $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} = 19$
 $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} = 99$

4

Rock is one of these numbers: 40, 41, 20, 21, 23, 24, 60, 63
 Rock is in this string picture.
 Who is Rock? 23

odd numbers
 multiples of 3
 Rock
 smaller than 40

5

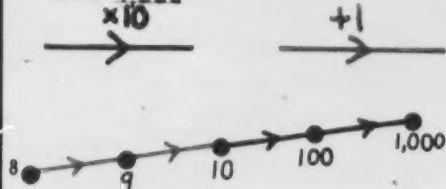
Complete the chart:

$\begin{array}{|c|c|} \hline \rightarrow & \rightarrow \\ \hline \times 2 & +4 \\ \hline \div 2 & +16 \text{ or } \times 5 \\ \hline \times 10 & -60 \text{ or } -4 \\ \hline \times 4 & -12 \\ \hline \div 4 & \times 10 \text{ or } +18 \\ \hline +20 & -8 \\ \hline \end{array}$
 $\begin{array}{|c|c|} \hline \rightarrow & \rightarrow \\ \hline +2 & \times 2 \\ \hline +4 \text{ or } -6 & \times 10 \\ \hline +17 & -5 \\ \hline +7 & +5 \\ \hline \times 5 \text{ or } +32 & \div 2 \\ \hline -3 & \times 4 \\ \hline \end{array}$

6

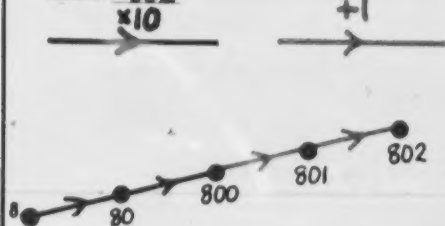
Ku is the largest possible ending number of a red-blue road starting at 8 and using exactly two red arrows and two blue arrows.

Who is Ku? **1,000**



Tu is the smallest possible ending number of a red-blue road starting at 8 and using exactly two red arrows and two blue arrows.

Who is Tu? **802**

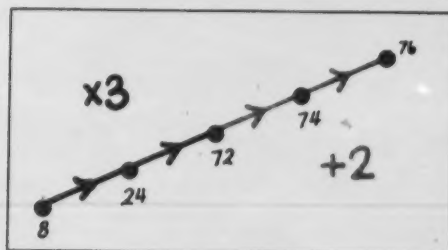
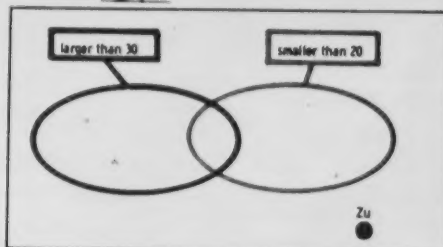


7

Zu is a secret number.

Zu is in this string picture and in this arrow picture.

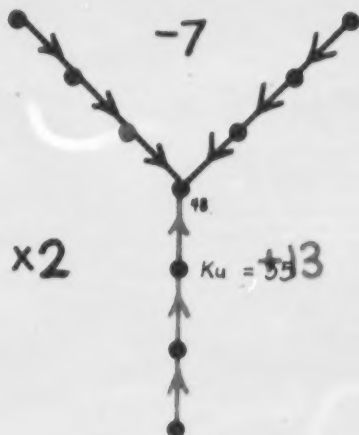
Who is Zu? **24**



8

Ku is the only multiple of 7 in this picture.

Where is Ku? Write its name on the picture.



9

Write these numbers on the Minicomputer with exactly three checkers.

You may use both positive and negative checkers.

$$\begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \bullet & \bullet \\ \hline \end{array} = 27$$

$$\begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \bullet & \bullet \\ \hline \end{array} = 59$$

$$\begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \bullet & \bullet \\ \hline \end{array} = 75$$

$$\begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \bullet & \bullet \\ \hline \end{array} = 87$$

$$\begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \bullet & \bullet \\ \hline \end{array} = 99$$

$$\begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \bullet & \bullet \\ \hline \end{array} = 120$$

$$\begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \bullet & \bullet \\ \hline \end{array} = 33$$

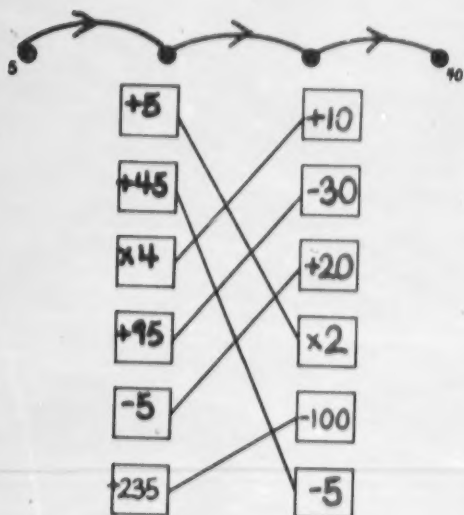
$$\begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \bullet & \bullet \\ \hline \end{array} = 200$$

$$\begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \bullet & \bullet \\ \hline \end{array} = 130$$

$$\begin{array}{|c|c|} \hline \bullet & \bullet \\ \hline \bullet & \bullet \\ \hline \end{array} = 73$$

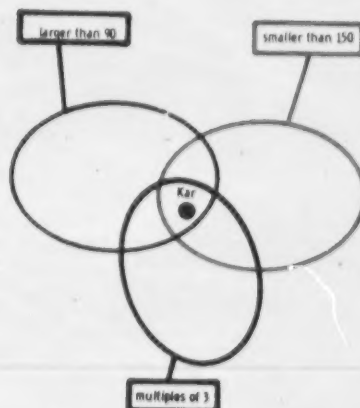
10

Pair the tags:



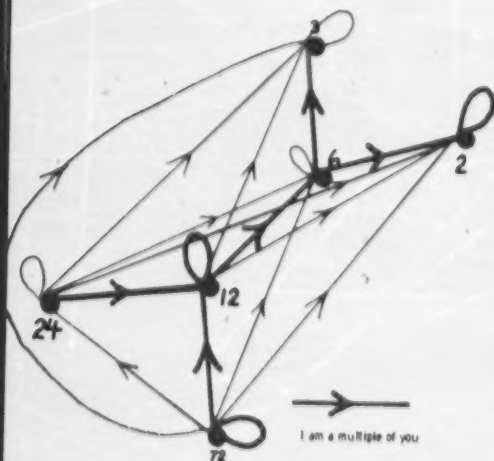
11

Kar is a secret number.
 Kar can be shown on the Minicomputer using exactly two positive
 checkers on the tens' board.
 Kar is in this string picture.
 Who is Kar? 120



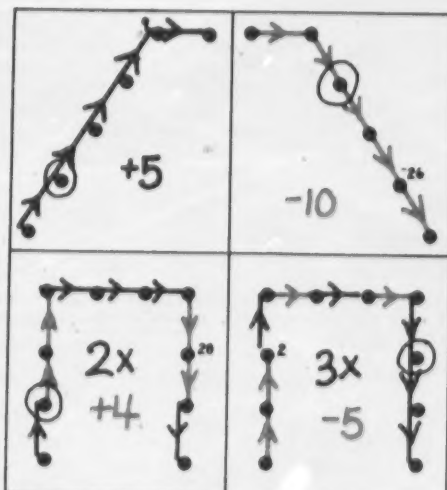
12

Label the dots. Many answers are possible.
 Then draw all the missing red arrows.
 Remember: You cannot have two dots for the same number.



13

6 is in each of these pictures.
 In each of them, circle its dot.

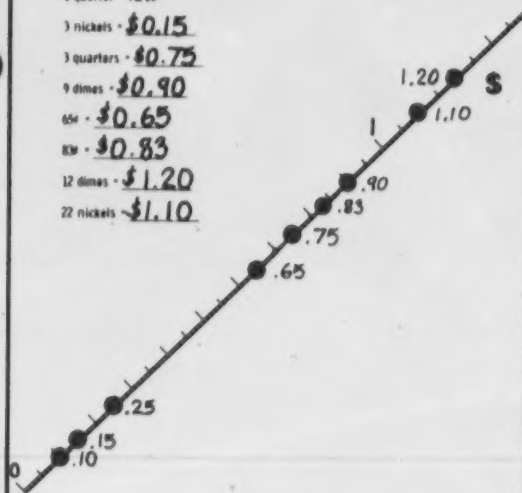


14

On this \$ number line, put dots for:

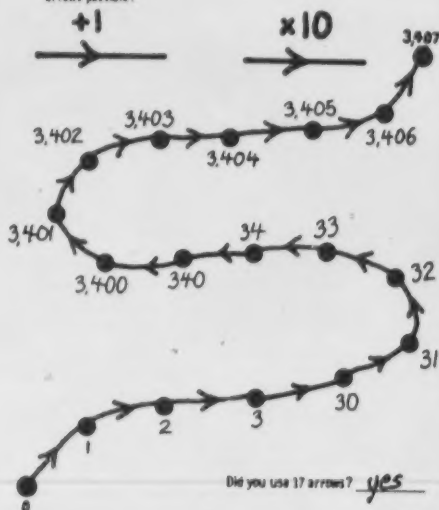
1 dime = \$0.10

1 quarter = \$0.25

3 nickels = \$0.153 quarters = \$0.759 dimes = \$0.906¢ = \$0.068¢ = \$0.0812 dimes = \$1.2022 nickels = \$1.10

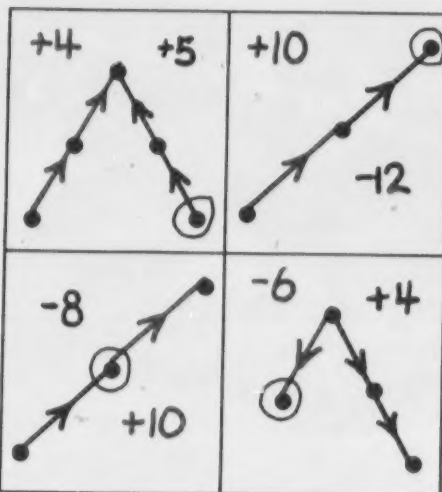
15

Build a blue-red road from 0 to 3,407 using the smallest number of arrows possible.

Did you use 37 arrows? yes

16

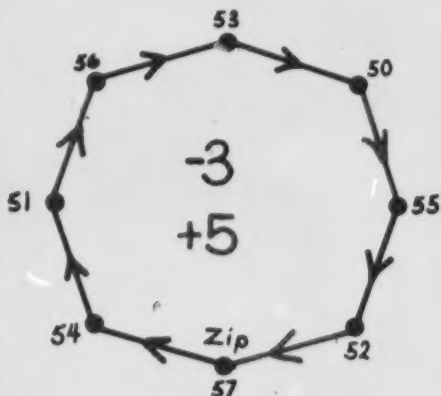
Circle the smallest number in each of the pictures:



A Collection of Problems #2 ***

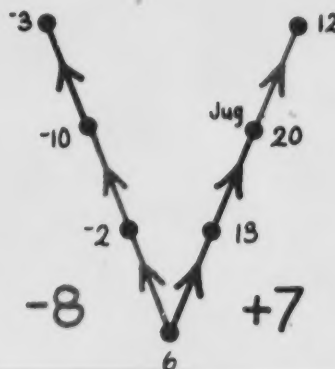
2

In this picture 50 is the smallest number and Zip is the largest number.
Who is Zip? 57



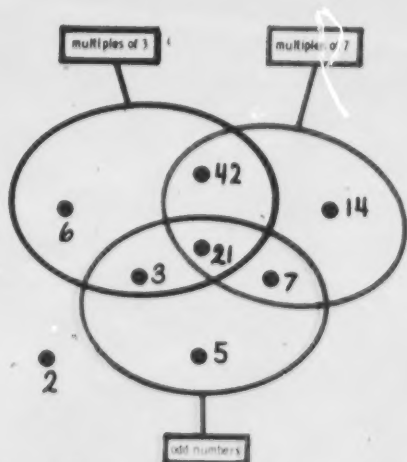
3

In this picture, 10 is the smallest number and Jug is the largest number.
Who is Jug? 20



4

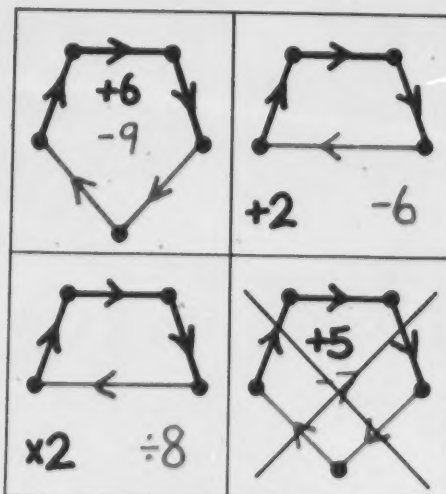
Label each dot. Many answers are possible.



(Many other solutions are possible.)

5

One of these pictures was drawn by a witch and is incorrect. Cross it out.



6

	=	<u>65.75</u>
	=	<u>111</u>
	=	<u>1.2</u>
	=	<u>1.1</u>
	=	<u>16.5</u>
	=	<u>.16</u>

7

Build a red-blue road from 100 to 1,000 using as few arrows as possible.

8

Lip is a secret number.
 Lip is the ending number of a red-blue road starting at 8 and using exactly two red arrows and two blue arrows.
 Lip can be put on the Minicomputer using exactly one positive checker.
 Who is Lip?

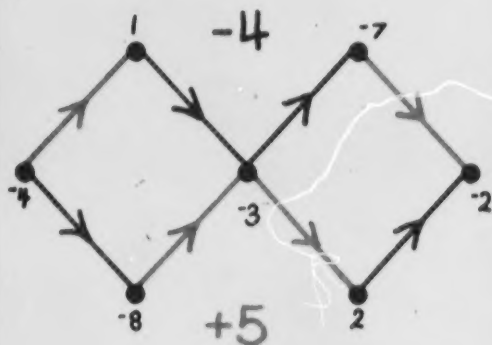
$\frac{2}{-5}$ $\frac{x2}{}$

9

	=	<u>8</u>		=	<u>0</u>
	=	<u>-6</u>		=	<u>-4</u>
	=	<u>-9</u>		=	<u>-49</u>
	=	<u>-36</u>		=	<u>135</u>
	=	<u>-48</u>		=	<u>-90</u>

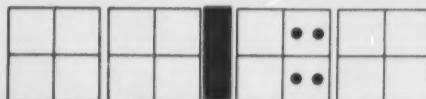
10

In this picture, Bin is the smallest number and 2 is the largest number.
Who is Bin? -8

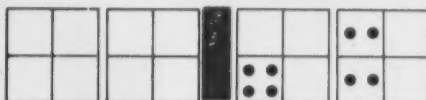


11

Put 1 on the Minicomputer using exactly four positive checkers.
Several answers are possible.



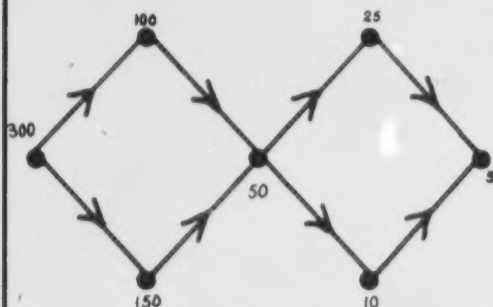
Put 1 on the Minicomputer using exactly eight positive checkers.
Several answers are possible.



(Other solutions are possible.)

12

Label the dots. Several answers are possible.
Then trace with your finger all the missing red arrows and count them.
Don't draw them.
How many red arrows are missing? 11 (plus the 7 loops)



(Other solutions are possible.)

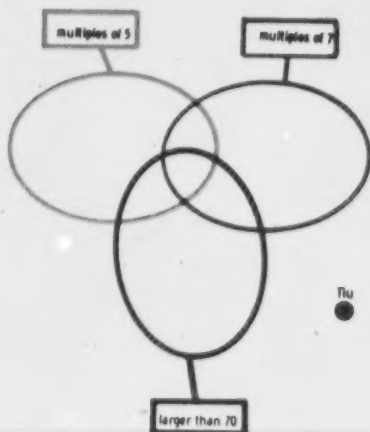
13

In each row of this chart, make change for \$1 with the number of coins indicated on the left.

Number of coins	Half-dollar .50	Quarter .25	Dime .10	Nickel .05	Penny .01
4		4			
5	1	1	2	1	
6		3	2	1	
7		2	5		
8		2	4	2	
9		1	7	1	
10			10		
12		2		10	
15			5	10	
20				20	
50		1	2	2	45

14

Tiu is a secret number larger than 68.
 Tiu cannot be shown on the Minicomputer with exactly three checkers
 (positive or negative).
 Tiu is in this string picture.
 Who is Tiu? 67



15

Here are some of Tiu's names. Complete them.

$$100 - \boxed{33}$$

$$1,000 - \boxed{933}$$

$$(2 \times \boxed{40}) - 13$$

$$75 - (\boxed{16} \div 2)$$

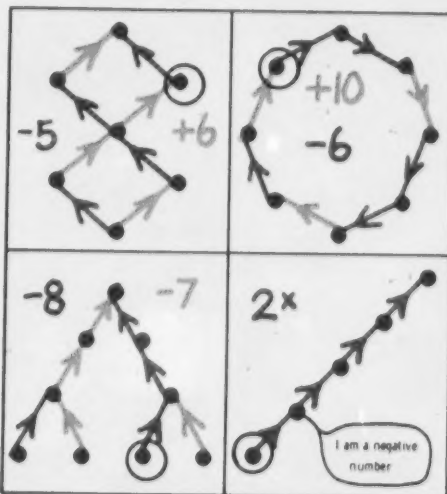
$$\boxed{134} \div 2$$

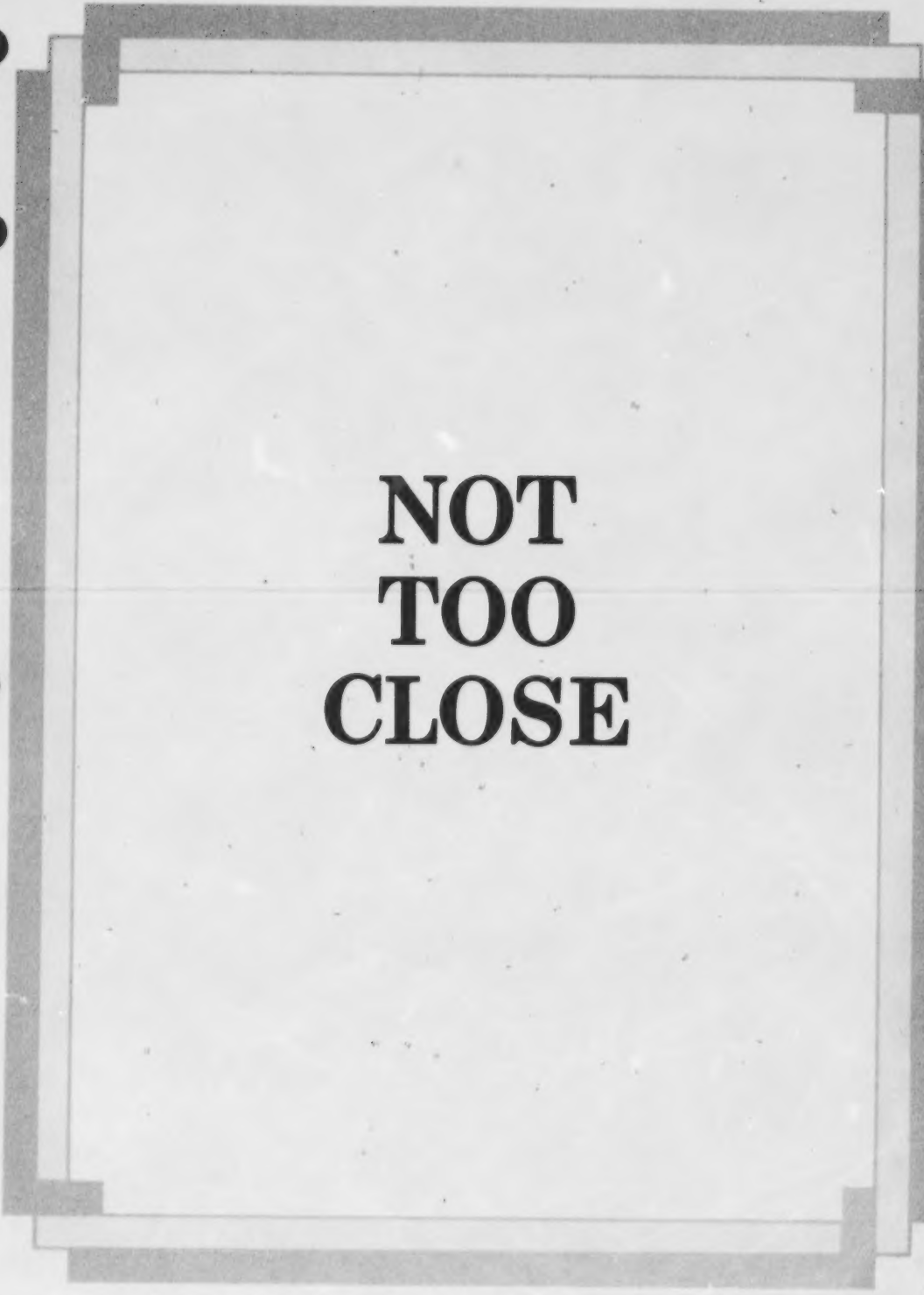
$$2 \times \boxed{33.5}$$

$$10 \times \boxed{6.7}$$

16

Circle the largest number in each of these pictures.





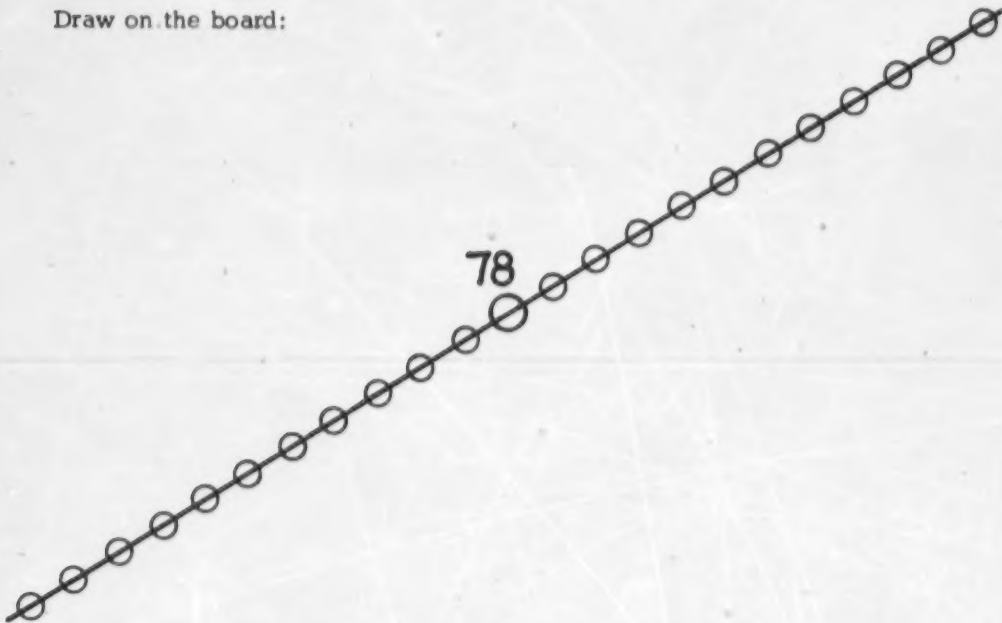
**NOT
TOO
CLOSE**

NOT TOO CLOSE

Lesson 1

Give a copy of the "Not Too Close" story-workbook to each student and ask them to read pages 3 to 5 and to answer the questions on page 5.

Draw on the board:



After a while, ask:

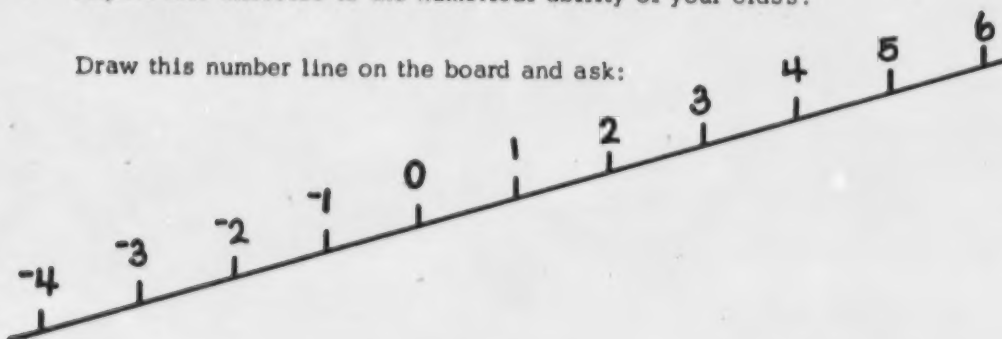
T: To which numbers will 78 throw a ball? (Answer: 73 and 83)
How do you know? (Possible answers: $83 = 78 + 5$ and $73 + 5 = 78$.
Or 83 is 5 more than 78 and 73 is 5 less than 78)

Ask the students to close their books for a few minutes and try to answer these questions:

T: What is the distance from 49 to 60? (Answer: 11)

Ask a similar question for some other pairs of positive numbers. For instance, (75; 84), (980; 1,001), (30; 3), (1,996; 2,003), (777; 888), (9,998; 10,005), (66; 101), (35; 35) and so on. Adjust this exercise to the numerical ability of your class.

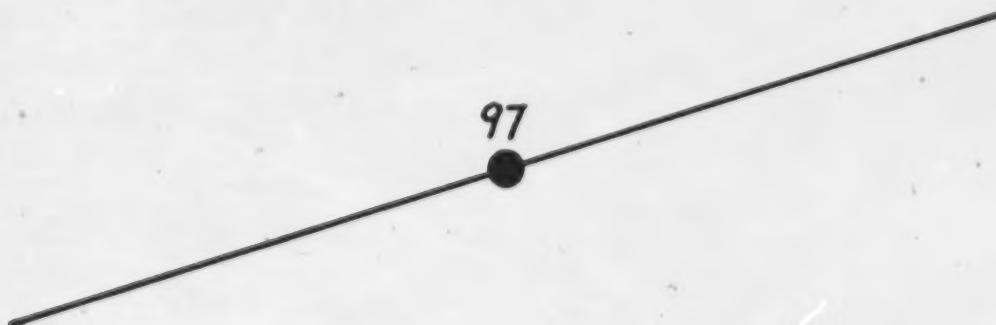
Draw this number line on the board and ask:



T: What is the distance from -3 to 5 ? (Answer: 8)
How do you know? (Answer: 5 is 8 more than -3)

Ask a similar question for some other pairs of positive and negative numbers. For instance, (-2 ; 2), (6; -1), (-3 ; 10), (-10 ; 15), (-12 ; 12) and so on.

Then have the students work independently on pages 6 to 10. After a while, draw this picture on the board:



T: To which numbers will 97 throw a ball? (Answer: 105 and 89)

Ask a student to put these two numbers on the number line.

T: Open your book to page 7. What is the largest number that you have colored green?

S: 15

T: How do you know?

S: Because 15 is 9 more than 6.

T: What is the smallest number that you have colored green?

S: -3

T: How do you know?

S: Because -3 is 9 less than 6

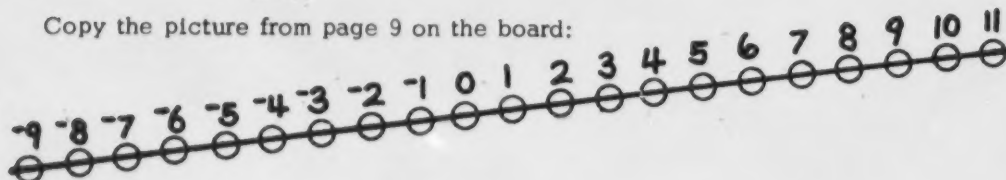
T: Which are the other numbers that you have colored green?

S: All the numbers between -3 and 15.

Be sure to discuss whether 6 should be colored. (It should be, because the distance from 6 to 6 is 0.)

Ask similar questions for the problem on page 8.

Copy the picture from page 9 on the board:



T: Open your books to page 9.

What are the dots we have to color green?

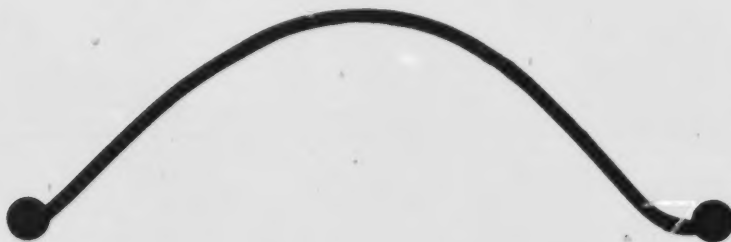
(Answer: 6; 7; 8; 9; 10; 11; and so on. In other words, all the numbers that are larger than 5.

-4; -5; -6; -7; -8; and so on. In other words, all the numbers that are smaller than -3.)

Ask the students to open their books to page 10 and to give many examples of pairs of numbers whose distance from each other is 6.

After they've given many such examples ask them to try and find all pairs with one positive and one negative number. [Answer: (-5; 1), (-4; 2), (-3; 3), (-2; 4), (-1; 5)]

Afterwards let the students work individually on pages 11 to 14. After a while, draw on the board:



T: In the game that is explained by the secret number, what does it mean for two dots to be joined by a red cord?

S: The distance between the two numbers has to be smaller than 10.

T: Let's check your answers to the questions on page 13. Could these dots be for 32 and 40?

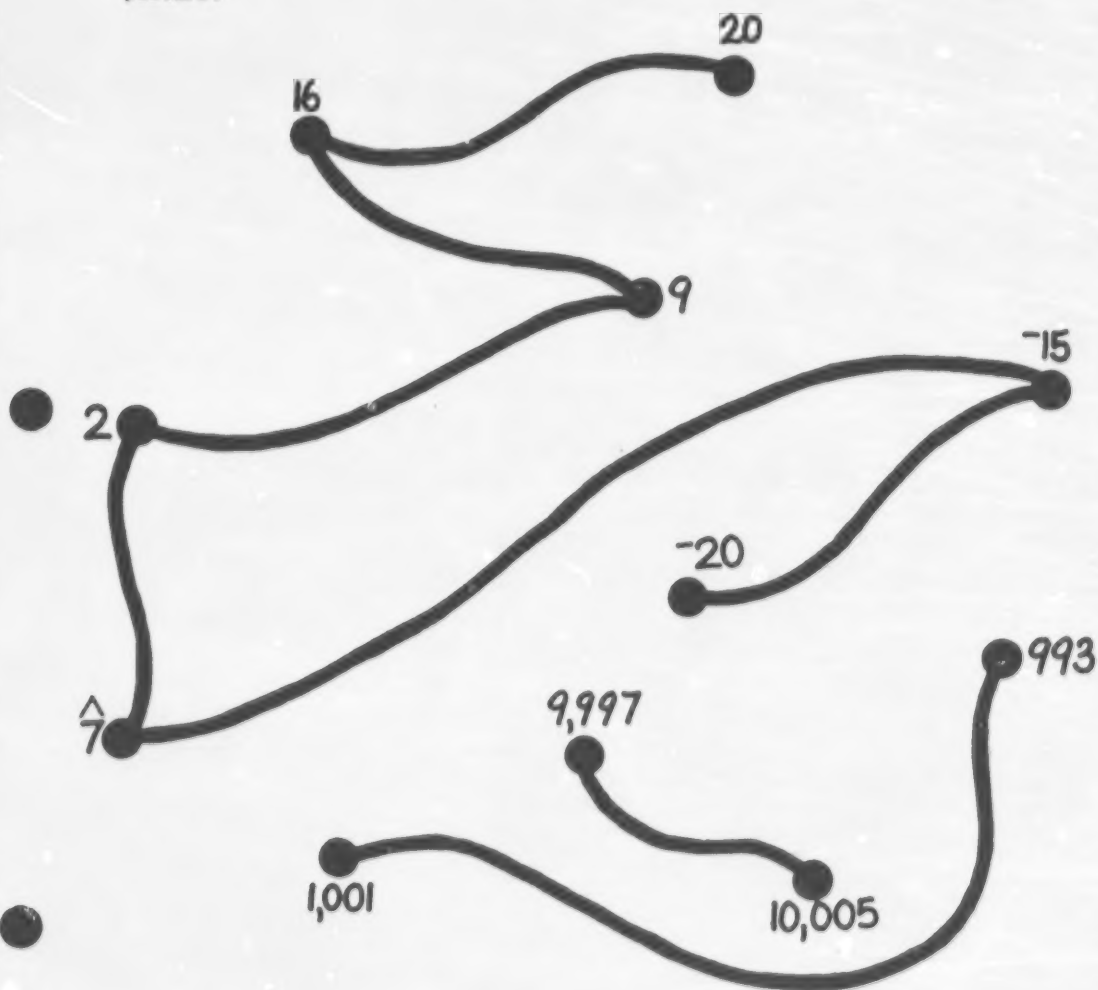
S: Yes.

T: How do you know? What is the distance between 32 and 40?

S: The distance is 8, and 8 is smaller than 10.

Follow a similar procedure for each of the other questions.

Ask the students to draw all the missing red cords on the picture on page 14. Copy this picture on the board and have students take turns to complete the picture.



NOT TOO CLOSE

Lesson 2

Exercise 1:

Draw on the board:

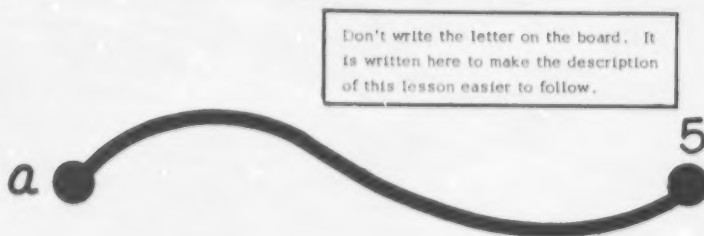


T: Do you remember the story of "Not Too Close" we were reading last week? Do you remember the rule of the red cords?

Encourage the students to explain the rule of the game.

T: Yes, two integers (positive or negative or zero) are joined by a red cord if and only if their distance is smaller than 10.

Ask the students to give you some examples of pairs of numbers that could be joined by a red cord. Write "5" on the picture:



Don't write the letter on the board. It is written here to make the description of this lesson easier to follow.

T: Which number could the other dot (a) be for?

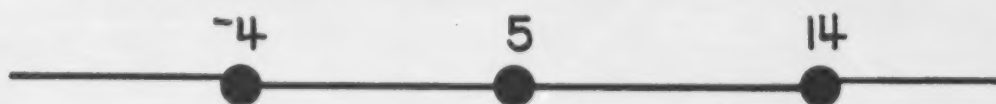
Let the students give some examples.

- T: What is the largest number that this dot (a) could be for?
(Answer: 14 because 14 is larger than 5 and the distance between 5 and 14 is 9)

Draw a number line and put a dot for 5 and 14.

- T: What is the smallest number that this dot (a) could be for?
(Answer: -4 because -4 is smaller than 5 and the distance between -4 and 5 is 9)

Put a dot for -4 on the number line.



Emphasize the fact that this dot (point to a) could be for -4 or 14 or any integer between them.

Exercise 2:

Draw on the board:



- T: Two numbers are joined by a blue cord if and only if their distance is 10 or larger than 10.

Write on the board:

Their distance is 10 or larger than 10.

Ask the students to give many examples of pairs of numbers that could be joined by a blue cord. When the calculation is at their level, ask them to indicate the distance between the numbers they are suggesting.

Write "100" on the picture and draw a number line:



Don't write the letter on the board. It is written here to make the description of this lesson easier to follow.



Point to b and ask:

T: On the number line, can you show where this number could be?
(Answer: 110 or to the right of 110; or 90 or to the left of 90)

Use green to color this part of the number line:



90

Exercise 3:

Give to each student his or her copy of "Not Too Close" and ask them to solve the problems on pages 16 and 17. On page 16, ask them to indicate the distance between each pair of numbers.

After a while, have students give their answers to the problems on page 16.

Draw on the board:

1,000

710

991

1,001

1,010

1

71

100

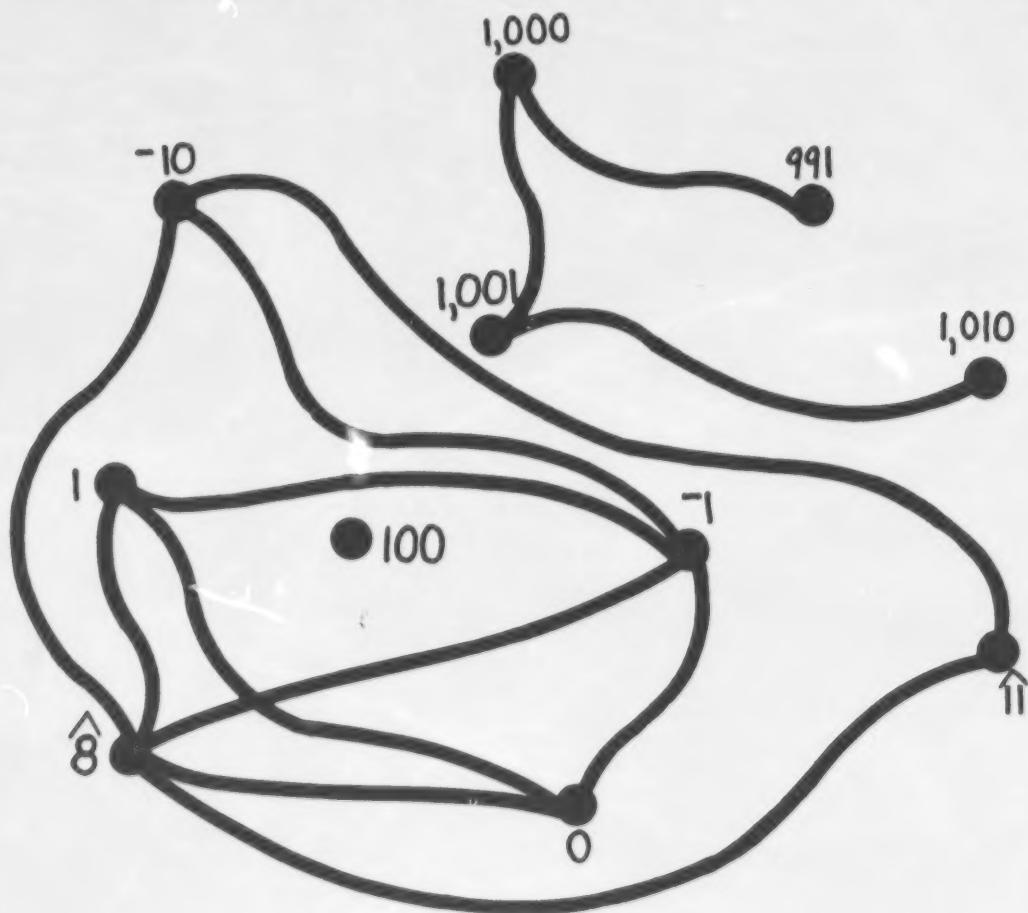
11

8

0

T: In your book, you have to draw all the blue cords in this picture.
On the board, I want you to draw all the red cords.

Have students take turns drawing all the red cords:



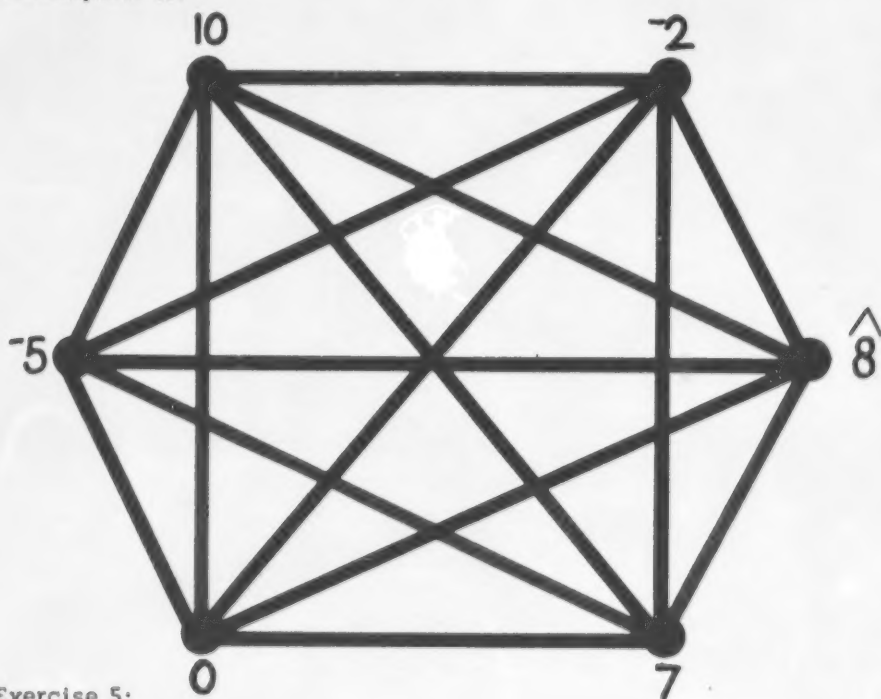
Ask the students to compare their blue picture to this red picture. Emphasize the fact that there is exactly one cord (blue or red) between each pair of numbers.

^{*}Don't worry if some students don't find all the cords. They should, however, understand that there are many.

Exercise 4:

T: Complete the picture on page 18.

Draw the picture from page 18 on the board and have students take turns to complete it.

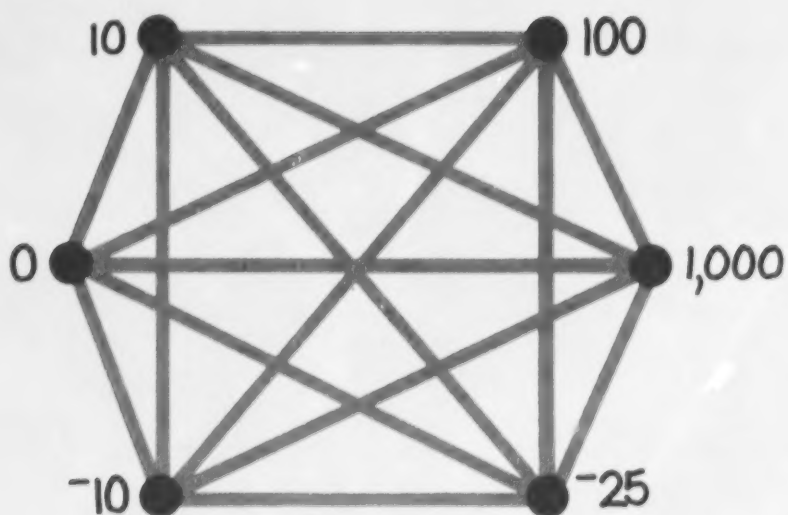


Exercise 5:

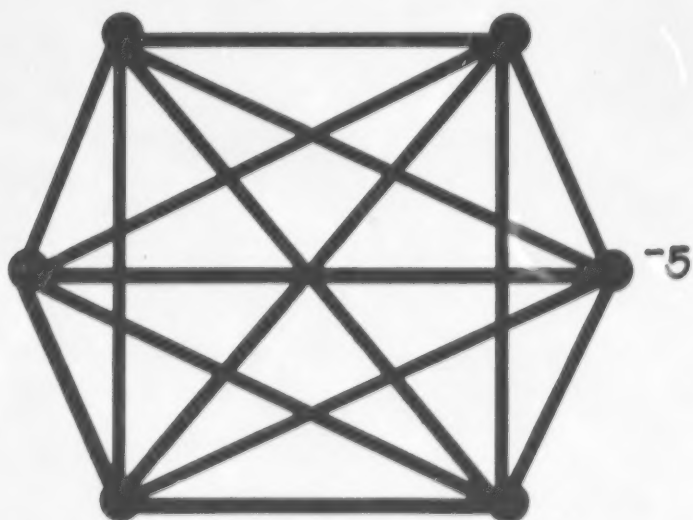
T: Label the dots for each picture on page 19. If you like, choose some positive and some negative numbers.

Let the students work on their own for a while. Draw the first picture on the board and have students take turns labeling the dots. Don't allow the students to copy this solution in their books. Each of them should have an original solution.

Example of solution:

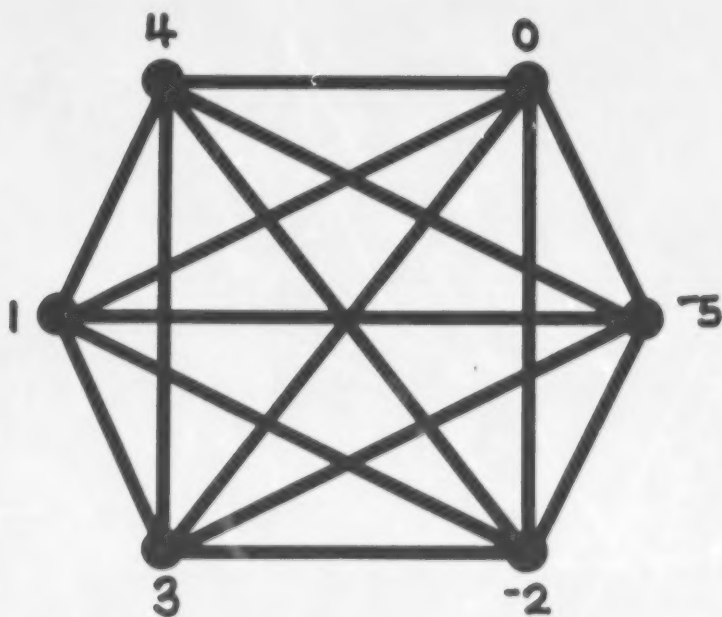


Draw the second picture from page 19 on the board and label one of its dots "-5".



Have the students take turns labeling the other dots.

Example of solution:



NOT TOO CLOSE

Lesson 3

Draw on the board:



T: Do you remember the story of "Not Too Close" we were reading last week? Do you remember the rule of the red cords?

Ask a student to explain it.

T: Yes, two integers (positive or negative or zero) are joined by a red cord if and only if their distance is smaller than 10. What number could be in the little box?

Encourage the students to give several answers (positive and negative).

T: What is the largest number this dot could be for?

S: 11

T: How do you know?

S: $2 + 9 = 11$

T: What is the smallest number this dot could be for?

S: $2 - 9 = -7$

T: So any integer from -7 to 11 could be in the little box.

Draw on the board:



Ask a student to tell us the rule of the blue cords. (Two integers are joined by a blue cord if and only if their distance is at least 10; in other words, if and only if their distance is 10 or larger than 10.)

T: What number could be in the little box?

Encourage the students to give many examples (positive and negative).

Draw the number line on the board:

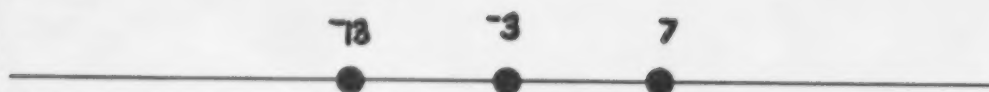


T: What is the smallest positive number that could be in the little box? (Answer: $-3 + 10 = 7$)

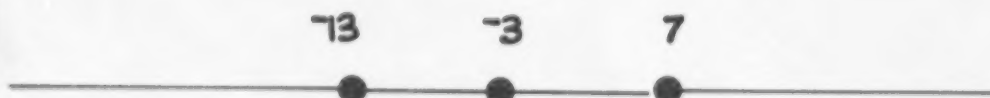
Put 7 on the number line.

T: What is the largest negative number that could be in the little box? (Answer: $-3 - 10 = -13$)

Put -13 on the number line.



Ask a student to trace with his or her finger all the places on the number line where the number (in the little box) could be. (Answer: starting from 7, all the dots for whole numbers to the right of 7; and starting from -13 , all the dots for negative integers to the left of -13 .) Draw this part of the number line in green.



Give each student his or her copy of "Not Too Close" and let them all work at their own pace.

Before the end of the period, ask the students if they have found which number is telling the story (pages 23 to 28). Have students who solved this problem explain their answer to the whole group.

Not Too Close

5

Looking at 78, Zero orders,
 "Throw a ball to each number whose distance from you is exactly 5."
 78 thinks for a while. Then he throws a ball to

73 and to 83

USE GREEN TO COLOR THE DOTS FOR THESE TWO NUMBERS.

(Shaded dots, shown in black on this page, should actually be in green on the students' pages.)

6

"Throw a ball to the numbers whose distance from you is exactly 8!"
 Zero orders 97.

COMPLETE THIS PICTURE SO THAT IT SHOWS WHAT 97 SHOULD DO.

7

"Everyone whose distance from 5 is less than 10, stand on your head!"
 commands Zero. They obey immediately.

USE GREEN TO COLOR THE DOTS FOR ALL THE NUMBERS WHO
 STAND ON THEIR HEADS.

(Shaded dots, shown in black on this page, should actually be in green on the students' pages.)

"Rest," says Zero.
 They stand on their feet again.

8

"Everyone whose distance from 2 is less than 7, stand on your head!"
 orders Zero.

USE GREEN TO COLOR THE DOTS FOR ALL THE NUMBERS
 WHO SHOULD STAND ON THEIR HEADS.

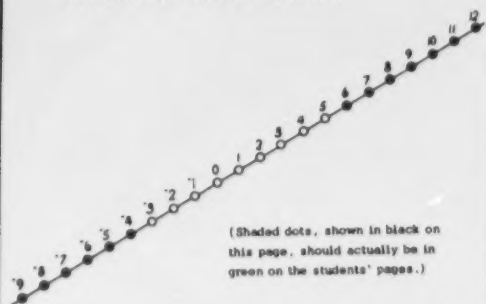
(Shaded dots, shown in black on this page, should actually be in green on the students' pages.)

"Rest," says Zero.
 They stand on their feet again.

9

Zero gives a similar order to all the numbers whose distance from 1 is more than 4.

USE GREEN TO COLOR THE DOTS FOR THESE NUMBERS.



These numbers are either larger than 5 or smaller than -3.
You weren't able to draw them all because there are too many of them.
But you can imagine all the rest.

10

"I need two numbers whose distance from each other is 6," says Zero.
Four pairs of numbers volunteered:

19 and 25

32 and 26

-4 and 2

-14 and -20

WRITE BELOW SOME OTHER PAIRS OF NUMBERS WHO COULD HAVE VOLUNTEERED.

1 and 79 and 1510 and 1611 and 170 and -6-3 and -9-15 and -215 and -1

(Many other solutions are possible.)

12

IN MY GAME

Two numbers are joined by a red cord
if and only if
the distance between them is less than 10.



WHICH NUMBERS COULD THESE DOTS BE FOR? THINK OF MANY
SOLUTIONS AND WRITE THEM ON A SEPARATE SHEET OF PAPER.

(Many solutions are possible: for example,
8 and 3; -2 and 4; -1 and 5 and so on.)

13



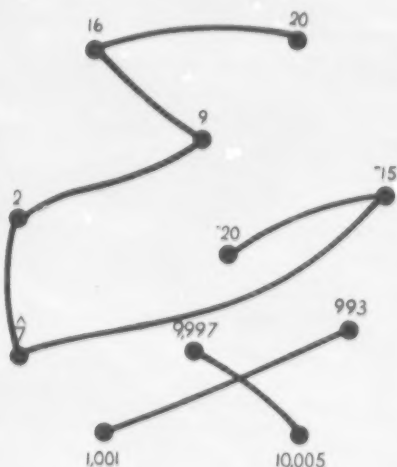
COULD THESE DOTS BE FOR

32 and 40? yes17 and 77? no95 and 104? yes998 and 1,006? yes-4 and 4? yes5 and -5? no4 and 74? no-4 and -74? no-3 and -14? yes

14

Here are some of my friends. They began to play my game but did not have the time to finish it.

CAN YOU DRAW THE REST OF THE RED CORDS FOR THEM?



15

IN MY GAME,

Two numbers are joined by a blue cord
if and only if
their distance is 10 or more than 10.



WHICH NUMBERS COULD THESE DOTS BE FOR? THINK OF MANY SOLUTIONS AND WRITE SOME OF THEM ON A SEPARATE SHEET OF PAPER.

(Many solutions are possible: for example, 18 and 156; "3 and "50 and so on.)

16

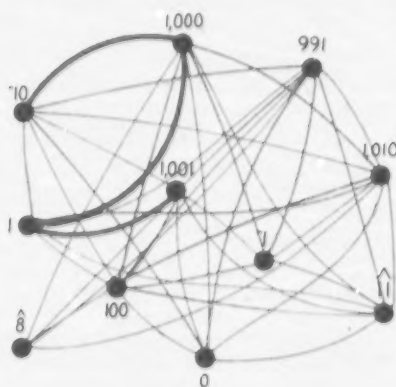


COULD THESE DOTS BE FOR

5 and 21?	yes
8 and 8?	yes
995 and 1,003?	no
8 and 1?	no
18 and 11?	no
18 and 11?	yes
9,995 and 10,005?	yes

17

On this picture, I have drawn three blue cords. Many more are missing. TRY TO DRAW THEM ALL.

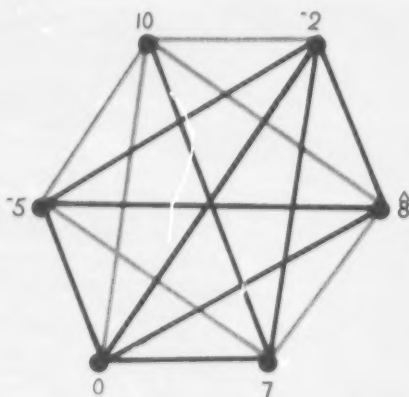


WE COULD ADD SOME RED CORDS TO THIS PICTURE. TRACE SOME OF THEM WITH YOUR FINGER, BUT DON'T DRAW THEM. WHAT DO YOU NOTICE?

(There is either a red cord or a blue cord between every two dots but never both.)

18

DRAW ALL THE RED CORDS AND ALL THE BLUE CORDS THAT ARE POSSIBLE BETWEEN THESE NUMBERS.

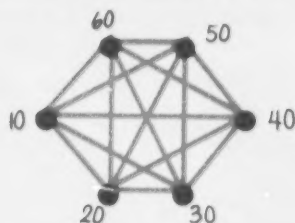


WHAT DO YOU NOTICE?

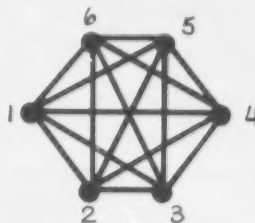
(Each number is connected to every other number by either a red or blue cord but never by both.)

(Many other solutions are possible.)

In the picture on page 18, some cords are red and some cords are blue. CAN YOU THINK OF SIX NUMBERS THAT CAN BE JOINED USING ONLY BLUE CORDS? WRITE THEIR NAMES ON THIS PICTURE.

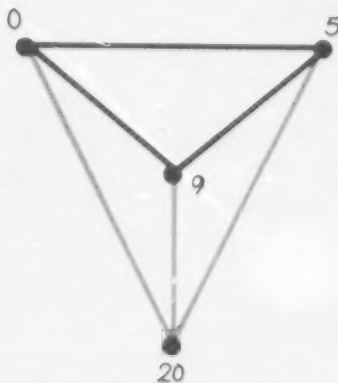


CAN YOU THINK OF SIX NUMBERS THAT CAN BE JOINED USING ONLY RED CORDS? WRITE THEIR NAMES ON THIS PICTURE.



20

CAN YOU THINK OF FOUR NUMBERS WHO COULD PLAY MY GAME IN THIS WAY? WRITE THEIR NAMES ON THE PICTURE.

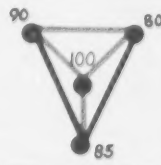


YOU COULD EASILY FIND MANY OTHER SOLUTIONS. DRAW SOME OF THEM ON A SEPARATE SHEET OF PAPER.

(Many other solutions are possible.)

21

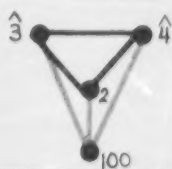
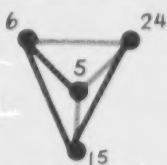
CAN YOU THINK OF CERTAIN NUMBERS WHO COULD PLAY IN THIS WAY? WRITE THEIR NAMES ON THESE PICTURES.



(Many other solutions are possible.)

22

AND WHAT ABOUT THESE FOUR PICTURES?
TRY TO LABEL THEIR DOTS.



DID YOU FIND THAT ONE OF THESE PICTURES IS WRONG?
IT'S IMPOSSIBLE TO PLAY THE GAME ACCORDING TO THE PICTURE IT
SHOWS. CROSS IT OUT.

(Many other solutions are possible.)

23

You still don't know who I am.
Now I shall give you some clues to help you discover who I am.

First clue:



Could I be 25? yes

Could I be 14? yes

Could I be 30? no

Could I be 0? no

Could I be 10? no

What is the largest number I could be? 29

What is the smallest number I could be? 11

24

Second clue:



Could I be 18? yes

Could I be 12? no

Could I be 29? yes

What is the smallest number I could be? 13

What is the largest number I could be? 29

Therefore I am larger than 12 and smaller than 30.

25

Third clue:



You already know that I am larger than 12 and smaller than 30.

Could I be 12? no

Could I be 13? yes

Could I be 14? yes

Could I be 20? no

Could I be 21? no

WRITE ALL THE NUMBERS THAT I COULD BE:

13, 14, 15, 16, 17, 18, 19.

26

Fourth clue:



You already know that I am one of these seven numbers:

~~13~~, ~~14~~, 15, 16, 17, 18, 19.

The fourth clue tells you that two of them are impossible.

CROSS THEM OUT.

27

Fifth clue:



Now you know that I am

15 or 16.

28

Sixth clue:

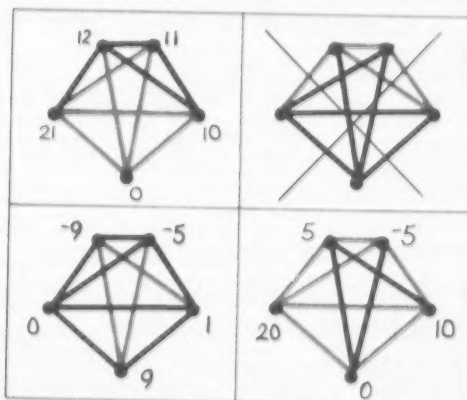


Have you found out who I am?

I am 15.

29

My friends played my game all day long. Very soon the walls of the school were covered with red and blue posters. Here are some of them. TRY TO LABEL THEIR DOTS.



Did you discover that one of these posters is wrong?
It's impossible to play the game according to the picture it shows.
Cross it out. (Many other solutions are possible.)

Master Schedule for the CSMP Intermediate Grades, Part I

M and F The World of Numbers **P** Probability, Statistics, and Combinatorics **W** Workbooks
G Geometry and Measurement **L** Languages of Strings and Arrows **A** Appendix to this supplement

1 M1.1 Transforming a Number and Arrow Pictures #1 M p. 4	2 L1.1 The String Game with A-Blocks L p. 2	3 W A Book About Me W p. 1	4 F1.2 Match the Tags #1 F p. 13	5 G1.1 Comparisons of Lengths and Areas G p. 3
6 M1.2 Transforming a Number and Arrow Pictures #2 M p. 18	7 L1.2 The String Game with Numbers: Multiples and Order L p. 12	8 W A Book About Me W p. 8	9 F2.1 Composition of Functions #1 F p. 42	10 G1.2 Introduction to Millimeters G p. 12
11 M2.1 Halving and Decimal Numbers M p. 54	12 L2.1 Logical Thinking #1 L p. 52	13 W A Book About Me W p. 10	14 F3.1 Guess My Rule #1 F p. 70	15 G1.3 Square Regions G p. 19
16 M3.1 Negative Integers on the Number Line and on the Minicomputer M p. 92	17 L2.2 Logical Thinking #2 L p. 61	18 W Collection of Problems 1 Tim and I W p. 23	19 F1.1 Tag the Arrow Pictures F p. 3	20 ADJUSTMENT DAY
21 M1.3 Introduction to the Minicomputer Golf Game M p. 27	22 G2.1 Circles #1 G p. 39	23 W Collection of Problems 1 Tim and I W p. 23	24 F4.1 Ten Times #1 F p. 94	25 String Game with A-Blocks
26 M4.1 Multiplication Problems #1 M p. 123	27 G1.4 Area Problems G p. 27	28 W Collection of Problems 1 Tim and I W p. 23	29 F5.1 Find the Secret Number F p. 117	30 P1.1 Same or Different #1 P p. 2
31 M3.2 Minicomputer Golf with Positive and Negative Checkers #1 M p. 99	32 G2.2 Circles #2 G p. 43	33 W Halloween Puzzles W p. 35	34 F4.2 Ten Times #2 F p. 106	35 P1.2 Same or Different #2 P p. 11
36 M2.2 Dollars, Dimes and Pennies M p. 64	37 ADJUSTMENT DAY	38 W Halloween Puzzles W p. 39	39 F2.2 Composition of Functions #2 F p. 49	40 P1.3 Same or Different #3 P p. 16

41 M5.1 Fractions #1 M p. 152	42 G2.3 Circles #3 G p. 47	43 W Halloween Puzzles W p. 45	44 F6.1 The Relation, "Is a Divisor of" #1 F p. 157	45 P1.4 Same or Different #4 P p. 22
46 M4.2 Multiplication Problems #2 M p. 133	47 M Minicomputer Golf with Positive and Negative Checkers #1 M p. 108	48 ADJUSTMENT DAY	49 F2.3 Composition of Functions #3 F p. 59	50 P1.5 Same or Different #5 P p. 30
51 M5.2 Fractions #2 M p. 159	52 L3.1 Composition Games #1 L p. 80	53 W Summer Camp W p. 53	54 F5.2 "Who Are We?" F p. 124	55 L2.3 Logical Thinking #3 L p. 67
56 M6.1 Division Problems #1 M p. 166	57 L3.2 Composition Games #2 L p. 90	58 W Summer Camp W p. 57	59 F1.3 Match the Tags #2 F p. 21	60 L2.4 Logical Thinking #4 L p. 72
61 M5.3 Fractions #3 M p. 164	62 L3.3 Composition Games #3 L p. 99	63 W Collection of Problems 2 W p. 69	64 F6.2 The Relation, "Is a Divisor of" #2 F p. 164	65 ADJUSTMENT DAY
66 M3.3 Minicomputer Golf with Positive and Negative Checkers #2 M p. 111	67 L3.4 Composition Games #4 L p. 105	68 W Collection of Problems 2 W p. 69	69 F5.3 Nix; P1f and Paf F p. 137	70 G2.4 Circles #4 G p. 51
71 ADJUSTMENT DAY	72 A Averages A p. 1	73 W Collection of Problems 2 W p. 69	74 F3.2 Guess My Rule #2 F p. 84	75 G2.5 The Triangle Inequality G p. 59
76 M2.3 Assorted Problems M p. 78	77 P2.1 "The Square Trap" #1 P p. 36	78 W Not Too Close W p. 83	79 F1.4 Find the Missing Arrows F p. 29	80 L1.3 The String Game with Numbers: Multiples, Divisors and Order #1 L p. 14
81 M4.3 Multiplication Problems #3 M p. 142	82 P2.2 "The Square Trap" #2 P p. 38	83 W Not Too Close W p. 88	84 F5.4 Nick and Nack F p. 147	85 L1.4 The String Game with Numbers: Multiples, Divisors and Order #2 L p. 21
86 M6.2 Division Problems #2 M p. 175	87 P2.3 "The Square Trap" #3 P p. 43	88 W Not Too Close W p. 96	89 ADJUSTMENT DAY	90 L1.5 The String Game with Numbers: Multiples, Divisors and Order #3 L p. 27